Discrete Mathematics
Chapter 1: Propositional and first order logic

GATE CS PYQ

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Section1: Engineering Mathematics

- **Discrete Mathematics**: Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations , generating functions.
- **Linear Algebra**: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.
- **Calculus**: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.
- **Probability and Statistics**: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

- **Ch 1:Mathematical Logic:**
- 1.1 Propositional Logic,
- 1.2 Propositional Equivalences,
- 1.3 Predicates and Quantifiers,
- 1.4 Nested Quantifiers,
- 1.5 Rules of Inference,
- 1.6 Introduction to Proofs.

- **GATE CS 2010 | Question: 30**
- Suppose the predicate $F(x,y,t)$ is used to represent the statement that person x can fool person y at time t.
- Which one of the statements below expresses best the meaning of the formula, $\forall x \exists y \exists t(\neg F(x,y,t))$

A. Everyone can fool some person at some time
B. No one can fool everyone all the time
C. Everyone cannot fool some person all the time
D. No one can fool some person at some time

- $F(x,y,t)$ = person x can fool person y at time t
- $\forall x \exists y \exists t(\neg F(x,y,t))$ take negation outside .
- $\neg \exists x\ \forall y\ \forall t(F(x,y,t))$ [By applying De Morgan's law.]
- Which means No one can fool everyone all the time
- Ans : B

- **GATE CS 2011 | Question: 30**
- Which one of the following options is CORRECT given three positive integers x,y and z, and a predicate **P(*x*)= $\neg(x=1) \wedge \forall y(\exists z(x=y*z) \Rightarrow (y=x) \vee (y=1))$**

A. **P(*x*)** being true means that x is a prime number

B. **P(*x*)** being true means that x is a number other than 1

C. **P(*x*)** is always true irrespective of the value of v

D. **P(*x*)** being true means that x has exactly two factors other than 1 and x

- **Statement:** $x \neq 1$ and if there exists some z for all y such that x=y*z , then y is either the number itself or 1.
- **This is the definition of prime numbers.**
- **Ans : (A) P(*x*)** being true means that x is a prime number.

- **GATE CS 2012 | Question: 1**
- Consider the following logical inferences.
- I_1: If it rains then the cricket match will not be played.
- The cricket match was played.
- Inference: There was no rain.
- I_2: If it rains then the cricket match will not be played.
- It did not rain.
- Inference: The cricket match was played.
- Which of the following is **TRUE**?

A. Both I_1 and I_2 are correct inferences
B. I_1 is correct but I_2 is not a correct inference
C. I_1 is not correct but I_2 is a correct inference
D. Both I_1 and I_2 are not correct inferences

- Let p : It rains ,q : cricket match will played.
- $I_1 : p \rightarrow \sim q$
- $\underline{\quad q}$
- $\therefore \sim p$
- I_1 is correct since it is in the form of Modus Tollens
- $I_2 : p \rightarrow \sim q$
- $\underline{\quad \sim p}$
- $\therefore q$
- It is false.
- If p false then q can be true or false.
- So I2 is incorrect inference.
- Ans: (B) I_1 is correct but I_2 is not a correct inference

- **GATE CS 2012 | Question: 13**
- What is the correct translation of the following statement into mathematical logic?
- "Some real numbers are rational"

A. $\exists x(\text{real}(x) \lor \text{rational}(x))$

B. $\forall x(\text{real}(x) \rightarrow \text{rational}(x))$

C. $\exists x(\text{real}(x) \land \text{rational}(x))$

D. $\exists x(\text{rational}(x) \rightarrow \text{real}(x))$

- Meaning of every statement :
- (A)There exists a number which is either real or rational
- (B)If a number is real it is rational
- (C)There exists some numbers which are real and rational
- (D)There exists a number such that if it is rational, it is real
- Ans : (C) $\exists x(\text{real}(x) \land \text{rational}(x))$

- **GATE CS 2013 | Question: 27**
- What is the logical translation of the following statement?
- "None of my friends are perfect."
- (A)$\exists x(F(x) \wedge \neg P(x))$ (B)$\exists x(\neg F(x) \wedge P(x))$
- (C)$\exists x(\neg F(x) \wedge \neg P(x))$ (D)$\neg \exists x(F(x) \wedge P(x))$
- Let F(x) : x is my friend ,P(x) x is perfect.
- (A) some of my friends are not perfect
- (B) some of those who are not my friends are perfect
- (C) some of those who are not my friends are not perfect
- (D) none of my friends are perfect
- $\forall x\ (F(x) \rightarrow \neg P(x))$
- $\forall x\ (\neg F(x) \vee \neg P(x))$
- $\forall x\ \neg (F(x) \wedge P(x))$
- $\neg \exists x(F(x) \wedge P(x))$
- Ans :(D)$\neg \exists x(F(x) \wedge P(x))$

- **GATE CS 2013 | Question: 47**
- Which one of the following is **NOT** logically equivalent to $\neg\exists x(\forall y(\propto)\wedge\forall z(\beta))$?
- (A) $\forall x(\exists z(\neg\beta)\rightarrow\forall y(\propto))$ (B) $\forall x(\forall z(\beta)\rightarrow\exists y(\neg\propto))$
- (C) $\forall x(\forall y(\propto)\rightarrow\exists z(\neg\beta))$ (D) $\forall x(\exists y(\neg\propto)\rightarrow\exists z(\neg\beta))$
- $\neg\exists x(\forall y(\propto)\wedge\forall z(\beta)) = \forall x\neg(\forall y(\propto)\wedge\forall z(\beta))$
- $=\forall x(\neg\forall y(\propto)\vee\neg\forall z(\beta))=\forall x(\forall y(\propto)\rightarrow\neg\forall z(\beta))$
- $=\forall x(\forall y(\propto)\rightarrow\exists z(\neg\beta))$ option (C) not (D)
- Now if we switch $\forall z(\beta)$ & $\forall y(\propto)$
- $=\forall x(\neg\forall z(\beta)\vee\neg\forall y(\propto))=\forall x(\forall z(\beta)\rightarrow\neg\forall y(\propto))$
- $=\forall x(\forall z(\beta)\rightarrow\exists y(\neg\propto))$ option (B) not (A)
- Ans : (A) & (D) are not logically equivalent

- **GATE CS 2014 Set 1 | Question: 1**
- Consider the statement "Not all that glitters is gold”
  Predicate glitters(x) is true if x glitters and predicate gold(x) is true if x is gold. Which one of the following logical formulae represents the above statement?
- (A) $\forall x$:glitters(x)$\Rightarrow\neg$gold(x) (B) $\forall x$:gold(x)$\Rightarrow$glitters(x)
- (C) $\exists x$:gold(x)$\wedge\neg$glitters(x) (D) $\exists x$:glitters(x)$\wedge\neg$gold(x)
- **"Not all that glitters is gold”** can be expressed as : $\neg(\forall x(\text{glitters}(x) \Longrightarrow \text{gold}(x)))$
- $\neg(\forall x(\text{glitters}(x) \Longrightarrow \text{gold}(x))) = \neg(\forall x(\neg\text{glitters}(x) \vee \text{gold}(x)))$
- $= \exists x\neg\ (\neg\text{glitters}(x) \vee \text{gold}(x)) = \exists x\ (\text{glitters}(x) \wedge \neg\text{gold}(x))$
- **"Not all that glitters is gold"** means **"some glitters are not gold"**
- (A) All that glitters are not gold
- (B) All golds are glitters
- (C) Some golds are not glitters
- (D) some glitters are not gold
- Ans: (D)$\exists x$:glitters(x)$\wedge\neg$gold(x)

- **GATE CS 2014 Set 1 | Question: 53**
- Which one of the following propositional logic formulas is TRUE when exactly two of p,q and r are TRUE?
- (A)$((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$ (B)$(\sim(p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
- (C)$((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$ (D)$(\sim(p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

p	q	r	A:$((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$	B:$(\sim(p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$	C:$((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$	D:$(\sim(p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$
T	T	F	T	T	T	F
T	F	T	F	T	F	F
F	T	T	F	T	T	F

- **Ans :** (B)$(\sim(p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

- **GATE CS 2014 Set 2 | Question: 53**
- Which one of the following Boolean expressions is NOT a tautology?
- (A) $((a\to b)\wedge(b\to c))\to(a\to c)$ (B) $(a\to c)\to(\sim b\to(a\wedge c))$
- (C) $(a\wedge b\wedge c)\to(c\vee a)$ (D) $a\to(b\to a)$
- (A) Let $(a\to c)$ =F,a=T,c=F
- b=T , $(a\to b)$=T, $(b\to c)$=F , $((a\to b)\wedge(b\to c))\to(a\to c)$=T
- b=F, $(a\to b)$=F, $(b\to c)$=T , $((a\to b)\wedge(b\to c))\to(a\to c)$=T ,tautology
- (B) Let $(\sim b\to(a\wedge c))$=F ,b=F, $(a\wedge c)$=F
- a=F,c=T, $(a\to c)\to(\sim b\to(a\wedge c))$ =F , a=F,c=F, $(a\to c)\to(\sim b\to(a\wedge c))$ =F, not tautology
- (C) Let $(c\vee a)$ = F ,c=F,a=F
- b =T/F , $(a\wedge b\wedge c)\to(c\vee a)$=T , tautology
- (D) Let $(b\to a)$=F , b=T ,a=F , $a\to(b\to a)$=T , tautology
- Ans : (B) $(a\to c)\to(\sim b\to(a\wedge c))$ is NOT a tautology

- **GATE CS 2014 Set 3 | Question: 1**
- Consider the following statements:
- P: Good mobile phones are not cheap Q: Cheap mobile phones are not good
- L: P implies Q M: Q implies P N: P is equivalent to Q
- Which one of the following about L, M, and N is CORRECT?
- (A)Only L is TRUE. (B)Only M is TRUE.
- (C)Only N is TRUE. (D)L, M and N are TRUE.
- Let G=good mobile phones ,C=cheap mobile phones.
- P: $(G \rightarrow \sim C) = (\sim G \vee \sim C)$
- Q: $(C \rightarrow \sim G) = (\sim C \vee \sim G) \cong P$
- L: $P \rightarrow Q = T$, M: $Q \rightarrow P = T$, N: $P \cong Q$
- Ans : (D)L, M and N are TRUE.

- **GATE CS 2014 Set 3 | Question: 53**
- The CORRECT formula for the sentence, "not all Rainy days are Cold" is
- (A) ∀d(Rainy(d)∧~Cold(d)) (B)∀d(~Rainy(d)→Cold(d))
- (C) ∃d(~Rainy(d)→Cold(d)) (D)∃d(Rainy(d)∧~Cold(d))
- ~ ∀d(Rainy(d)→Cold(d)) = ~ ∀d(~ Rainy(d) ∨Cold(d))
- ∃d ~ (~ Rainy(d) ∨Cold(d)) = ∃d (Rainy(d) ∧~Cold(d)) =Some Rainy days are not Cold.
- (A) All Rainy days are not Cold.
- (B) All days such that if its non Rainy then its Cold.
- (C) Some days such that if its non Rainy then its Cold.
- (D) Some Rainy days are not Cold.
- Ans : (D)∃d(Rainy(d)∧~Cold(d))

- **GATE CS 2015 Set 1 | Question: 14**
- Which one of the following is NOT equivalent to $p \leftrightarrow q$?
- (A) $(\neg p \lor q) \land (p \lor \neg q)$ (B) $(\neg p \lor q) \land (q \rightarrow p)$
- (C) $(\neg p \land q) \lor (p \land \neg q)$ (D) $(\neg p \land \neg q) \lor (p \land q)$
- $(p \leftrightarrow q) = (p \rightarrow q) \land (q \rightarrow p)$
- $= (\neg p \lor q) \land (q \rightarrow p)$ (B) As $(p \rightarrow q = \neg p \lor q)$
- $= (\neg p \lor q) \land (\neg q \lor p)$ (A)
- $= (\neg p \land (\neg q \lor p)) \lor (q \land (\neg q \lor p))$ [Distributive laws $p \land (q \lor r) = (p \land q) \lor (p \land r)$]
- $= (\neg p \land \neg q) \lor (\neg p \land p) \lor (q \land \neg q) \lor (q \land p)$
- $= (\neg p \land \neg q) \lor (p \land q)$ (D)
- Ans : (C) $(\neg p \land q) \lor (p \land \neg q)$ NOT equivalent to $p \leftrightarrow q$

- **GATE CS 2015 Set 1 | Question: 28**
- The binary operator ≠ is defined by the following truth table.
- Which one of the following is true about the binary operator ≠ ?
- (A) Both commutative and associative
- (B) Commutative but not associative
- (C) Not commutative but associative
- (D) Neither commutative nor associative
- (p ≠ q)=(q ≠ p) , Commutative
- Let p=0,q=1,r=0 ⇒(p ≠ q) ≠ r =1, p ≠ (q ≠ r)=1
- Let p=0,q=0,r=1 ⇒(p ≠ q) ≠ r =1, p ≠ (q ≠ r)=1 , Associative
- Its same as exclusive or ⊕
- Ans : (A) Both commutative and associative

p	q	p ≠ q
0	0	0
0	1	1
1	0	1
1	1	0

- **GATE CS 2015 Set 2 | Question: 3**
- Consider the following two statements.
- S_1: If a candidate is known to be corrupt, then he will not be elected
- S_2: If a candidate is kind, he will be elected
- Which one of the following statements follows from S_1 and S_2 as per sound inference rules of logic?
- (A) If a person is known to be corrupt, he is kind
- (B) If a person is not known to be corrupt, he is not kind
- (C) If a person is kind, he is not known to be corrupt
- (D) If a person is not kind, he is not known to be corrupt
- Let C: candidate is corrupt, E:he will be elected, K: candidate is kind .
- S_1=C→¬E= ¬ C ∨ ¬E
- S_2=K →E = ¬K∨E
- By **resolution**$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$.
- Conclusion = ¬C∨¬K
- C → ¬K or K → ¬C
- (A) C → K (B) ¬C → ¬K (C)K → ¬C (D) ¬K → ¬C
- Ans : (C) If a person is kind, he is not known to be corrupt

- **GATE CS 2015 Set 2 | Question: 55**
- Which one of the following well-formed formulae is a tautology?
- (A) $\forall x\ \exists y\ R(x,y) \leftrightarrow \exists y \forall x R(x,y)$
- (B) $(\forall x[\exists y\ R(x,y) \rightarrow S(x,y)]) \rightarrow \forall x \exists y\ S(x,y)$
- (C) $[\forall x \exists y (P(x,y) \rightarrow R(x,y))] \leftrightarrow [\forall x \exists y (\neg P(x,y) \lor R(x,y))]$
- (D) $\forall x \forall y P(x,y) \rightarrow \forall x \forall y P(y,x)$
- (A) $\forall x\ \exists y\ R(x,y) \not\Leftrightarrow \exists y \forall x R(x,y)$.We can't switch $\forall,\exists$ quantifier ,meaning will be change .
- Not tautology
- (B) When S=F,R=F, this implication will be False , not tautology .
- (C) Both are equivalent as $P \rightarrow Q = \sim P \lor Q$,Tautology .
- (D) $P(x,y) \neq P(y,x)$,Not tautology
- Ans : (C) $[\forall x \exists y (P(x,y) \rightarrow R(x,y))] \leftrightarrow [\forall x \exists y (\neg P(x,y) \lor R(x,y))]$

GATE CS 2015 Set 3 | Question: 24

- In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking the person replies the following
  "The result of the toss is head if and only if I am telling the truth"
- Which of the following options is correct?
- (A)The result is head (B)The result is tail
- (C) If the person is of Type 2, then the result is tail
- (D) If the person is of Type 1, then the result is tail
- Let H:The result of the toss is head ,T:I am telling the truth
- Case-1: Let the person be type 1. Type 1 always tells truth.
- $H \Leftrightarrow T$ =True ,T=True So H=true .The result is head.
- Case-2 : Let the person be type 2. Type 2 always tells lies.
- $H \Leftrightarrow T$ =False ,T=False ,H=True .The result is head.
- Ans : (A)The result is head

- **GATE CS 2016 Set 1 | Question: 1**
- Let *p*,*q*,*r*,*s* represents the following propositions.
- p : $x \in \{8,9,10,11,12\}$
- q :x is a composite number .
- r : x is a perfect square .
- s : x is a prime number.
- The integer x ≥2 which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ is ______________.
- $(p \Rightarrow q) = \{8, 9, 10, 12\}$
- $\neg r = \{8, 10, 11, 12\}$
- $\neg s = \{8, 9, 10, 12\}$
- $(\neg r \vee \neg s) = \{8, 9, 10, 11, 12\}$
- $(p \Rightarrow q) \wedge (\neg r \vee \neg s) = \{8, 9, 10, 12\}$
- $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s)) = 11$.
- Ans : 11

- **GATE CS 2016 Set 2 | Question: 01**
- Consider the following expressions:
- I.false II.Q III.true IV.P ∨Q V. ¬Q ∨P
- The number of expressions given above that are logically implied by P∧(P⇒Q) is ________.
- P∧(P⇒Q) =P∧(¬ P ∨ Q) =(P ∧ ¬ P) ∨(P ∧ Q) =F ∨(P ∧ Q) = (P ∧ Q)=X

P	Q	P ∧ Q=X	I.X⇒F	II.X⇒Q	III.X⇒T	(P∨Q)	IV.X⇒(P∨Q)	¬Q ∨P	V.X⇒(¬Q ∨P)
T	T	T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	F	T
F	F	F	T	T	T	F	T	T	T

- II.Q , III.true , IV.P ∨Q , V. ¬Q ∨P expressions are logically implied by P∧(P⇒Q) .
- Ans : 4

- **GATE CS 2017 Set 1 | Question: 01**
- The statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statements below?
- I. $p \Rightarrow q$ II. $q \Rightarrow p$ III. $(\neg q) \vee p$ IV. $(\neg p) \vee q$
- (A) I only (B) I and IV only (C) II only (D) II and III only
- I. $p \Rightarrow q \not\equiv (\neg p) \Rightarrow (\neg q)$
- II. $q \Rightarrow p \equiv (\neg p) \Rightarrow (\neg q)$ [contrapositive]
- III. $(\neg q) \vee p \equiv (\neg p) \Rightarrow (\neg q)$
- $(\neg p) \Rightarrow (\neg q) \equiv \neg (\neg p) \vee (\neg q) \equiv p \vee (\neg q)$
- IV. $(\neg p) \vee q \not\equiv (\neg p) \Rightarrow (\neg q)$
- Ans : (D) II and III only

- **GATE CS 2017 Set 1 | Question: 02**
- Consider the first-order logic sentence $F:\forall x(\exists y R(x,y))$. Assuming non-empty logical domains, which of the sentences below are *implied* by *F*?
- I. $\exists y(\exists x R(x,y))$ II. $\exists y(\forall x R(x,y))$ III. $\forall y(\exists x R(x,y))$ IV. $\neg\exists x(\forall y \neg R(x,y))$
- (A) IV only (B) I and IV only (C) II only (D) II and III only
- $\forall x(\exists y R(x,y)) \rightarrow \exists x(\exists y R(x,y)) = \exists y(\exists x R(x,y))$,I. true.
- II. We can't switch different quantifiers .False
- III. We can't derive existential to universal quantifiers.False
- IV. $\neg\exists x(\forall y \neg R(x,y)) = \forall x \neg(\forall y \neg R(x,y))$
- $= \forall x (\exists y R(x,y)) = F$. True
- Ans : (B)I and IV only

- **GATE CS 2017 Set 1 | Question: 29**
- Let p, q and r be propositions and the expression (p→q)→r be a contradiction. Then, the expression (r→p)→q is
- (A)a tautology
- (B)a contradiction
- (C)always TRUE when p is FALSE
- (D)always TRUE when q is TRUE
- The expression (r → p) → q is always true when q is true, regardless of the value of the expression (p → q) → r.
- Or
- Given (p → q) → r is false, then (p → q) is true and r is false.
- The possible cases are I-(p:T, q:T, r:F),II-(p:F, q:T, r:F),III-(p:F, q:F, r:F)
- **(A)** For (p:F, q:F, r:F), (r → p) → q is false . It is not a tautology
- **(B)** For case (i) and case (ii), (r → p) → q is true.It is not a contradiction .
- **(C)** For (p:F, q:F, r:F), (r → p) → q is also false.
- **(D)** (r → p) → q always TRUE when q is TRUE
- Ans : (D)always TRUE when q is TRUE

- **GATE CS 2017 Set 2 | Question: 11**
- Let p,q,r denote the statements "*It is raining*", "*It is cold*", and "*It is pleasant*", respectively. Then the statement "*It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold*" is represented by
- (A) $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$
- (B) $(\neg p \wedge r) \wedge ((p \wedge q) \rightarrow \neg r)$
- (C) $(\neg p \wedge r) \vee ((p \wedge q) \rightarrow \neg r)$
- (D) $(\neg p \wedge r) \vee (r \rightarrow (p \wedge q))$
- *It is not raining and it is pleasant* : $(\neg p \wedge r)$.
- *It is raining and it is cold* :$(p \wedge q)$
- *It is not pleasant only if it is raining and it is cold* : $\neg r \rightarrow (p \wedge q)$
- "*It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold*" : $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$
- Ans : (A)$(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$

GATE CS 2018 | Question: 28

- Consider the first-order logic sentence
- $\varphi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \ \psi(s,t,u,v,w,x,y)$
- where $\psi(s,t,u,v,w,x,y)$ is a quantifier-free first-order logic formula using only predicate symbols, and possibly equality, but no function symbols. Suppose φ has a model with a universe containing 7 elements.
- Which one of the following statements is necessarily true?
- (A) There exists at least one model of φ with universe of size less than or equal to 3
- (B) There exists no model of φ with universe of size less than or equal to 3
- (C) There exists no model of φ with universe size of greater than 7
- (D) Every model of φ has a universe of size equal to 7
- Let $\psi(s,t,u,v,w,x,y) = s+t+u+v+w+x+y>100$
- Now $\varphi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \ s+t+u+v+w+x+y>100$
- Let s=40 , t=50 , u=60 , v ,w ,x ,y can be any number .
- Even s=t=u =100
- Ans: (A) There exists at least one model of φ with universe of size $\leq$ 3

- **GATE CS 2019 | Question: 35**
- Consider the first order predicate formula φ:
- $\forall x[(\forall z\ z|x \Rightarrow ((z=x) \vee (z=1))) \rightarrow \exists w(w>x) \wedge (\forall z\ z|w \Rightarrow ((w=z) \vee (z=1)))]$
- Here a|b denotes that 'a divides b', where a and b are integers. Consider the following sets:
- S_1:{1,2,3,…,100} S_2: Set of all positive integers S_3: Set of all integers
- Which of the above sets satisfy φ?
- (A) S_1 and S_2 (B) S_1 and S_3 (C) S_2 and S_3 (D) S_1, S_2 and S_3
- Let P=$(\forall z\ z|x \Rightarrow ((z=x) \vee (z=1)))$,x is a prime number.
- Q=$\exists w(w>x)$,w is greater than x.
- R=$(\forall z\ z|w \Rightarrow ((w=z) \vee (z=1)))$ w is a prime number.
- P→Q∧R
- It simply says that if x is a prime number in the set then there exists another prime number w in the set which is larger than x .
- S_1:it can not be satisfy in **finite**, like 97 ∈ S_1 ,No w greater than 97.
- Only sets S_2 and S_3 satisfy φ.
- Ans : (C) S_2 and S_3

- **GATE CS 2020 | Question: 39**
- Which one of the following predicate formulae is NOT logically valid?
- Note that W is a predicate formula without any free occurrence of x.
- (A)$\forall x(p(x) \lor W) \equiv \forall x p(x) \lor W$ (B)$\exists x(p(x) \land W) \equiv \exists x p(x) \land W$
- (C)$\forall x(p(x) \rightarrow W) \equiv \forall x p(x) \rightarrow W$ (D)$\exists x(p(x) \rightarrow W) \equiv \forall x p(x) \rightarrow W$
- (A)It's valid as W is free. $\forall x$ can be associated with p(x)
- (B) It's valid as W is free. $\exists x$ can be associated with p(x)
- (C) $\forall x(p(x) \rightarrow W)$ $= \forall x(\neg p(x) \lor W)$
- $= \forall x \neg p(x) \lor W$ $= \neg \exists x p(x) \lor W$
- $= \exists x p(x) \rightarrow W \neq \forall x p(x) \rightarrow W$ it's not valid
- (D) $\exists x(p(x) \rightarrow W)$ $= \exists x(\neg p(x) \lor W)$
- $= \exists x \neg p(x) \lor W$ $= \neg \forall x p(x) \lor W$
- $= \forall x p(x) \rightarrow W$ =RHS it's valid
- Ans : (C)$\forall x(p(x) \rightarrow W) \equiv \forall x p(x) \rightarrow W$

- **GATE CS 2021 Set 1 | Question: 7**
- Let p and q be two propositions. Consider the following two formulae in propositional logic.
- S_1:(¬p∧(p∨q))→q S_2:q→(¬p∧(p∨q))
- Which one of the following choices is correct?
- (A) Both S_1 and S_2 are tautologies.
- (B) S_1 is a tautology but S_2 is not a tautology
- (C) S_1 is not a tautology but S_2 is a tautology
- (D) Neither S_1 nor S_2 is a tautology
- If consequence is false and hypothesis is true, then we will get False in the truth table.
- (¬p∧(p∨q))→q [Let q false ,Try to make (¬p∧(p∨q)) true]
- If p=F (T ∧ (F∨F))→F it's true, or
- If p=T (F ∧ (T∨F))→F it's true
- S_1 is a tautology
- q→(¬p∧(p∨q)) [Let q true try to make (¬p∧(p∨q)) false]
- If p=T T →(F ∧(T∨T)) it's false, or
- If p=F T →(T ∧(F∨T)) it's true
- S_2 is not a tautology
- Ans : (B) S_1 is a tautology but S_2 is not a tautology

- $S_1:(\neg p \wedge (p \vee q)) \rightarrow q$ $S_2: q \rightarrow (\neg p \wedge (p \vee q))$

p q	¬p	p∨q	(¬p∧(p∨q))	$S_1:(\neg p \wedge (p \vee q)) \rightarrow q$	$S_2: q \rightarrow (\neg p \wedge (p \vee q))$
T T	F	T	F	T	F
T F	F	T	F	T	T
F T	T	T	T	T	T
F F	T	F	F	T	T

- Ans : (B) S_1 is a tautology but S_2 is not a tautology

- ***GATE CS 2021 Set 2 | Question: 15**
- Choose the correct choice(s) regarding the following proportional logic assertion S:
- $S: ((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$
- (A) S is neither a tautology nor a contradiction
- (B) S is a tautology (C) S is a contradiction
- (D)The antecedent of S is logically equivalent to the consequent of S.
- Antecedent of S : $((P \wedge Q) \rightarrow R)$
- $= \sim(P \wedge Q) \vee R$
- $= \sim P \vee \sim Q \vee R$
- Consequent of S : $((P \wedge Q) \rightarrow (Q \rightarrow R))$
- $= \sim(P \wedge Q) \vee (Q \rightarrow R))$
- $=(\sim P \vee \sim Q) \vee (\sim Q \vee R))$
- $= \sim P \vee \sim Q \vee R$
- Both are logically equivalent .So there implication will be true .
- Ans : (B) ,(D)

P Q R	P∧Q	(P∧Q)→R	Q → R	((P∧Q)→(Q → R))	((P∧Q)→R) → ((P∧Q)→(Q → R))
T T T	T	T	T	T	T
T T F	T	F	F	F	T
T F T	F	T	T	T	T
T F F	F	T	T	T	T
F T T	F	T	T	T	T
F T F	F	T	F	T	T
F F T	F	T	T	T	T
F F F	F	T	T	T	T

- Ans : (B) S is a tautology
- (D)The antecedent of S is logically equivalent to the consequent of S.

- ***GATE CS 2023 | Question: 16**
- Geetha has a conjecture about integers, which is of the form $\forall x(P(x)\rightarrow\exists y\, Q(x,y))$
- Where P is a statement about integers, and Q is a statement about pairs of integers. Which of the following (one or more) option(s) would imply Geetha's conjecture?
- (a) $\exists x (P(x) \land \forall y\, Q(x,y))$ (b) $\forall x \forall y\, Q(x,y)$
- (c) $\exists y \forall x (P(x)\rightarrow Q(x,y))$ (d) $\exists x (P(x) \land \exists y Q(x,y))$
- L : "For every x if P(x) is true then there exists some y such that Q(x,y) will be true.
- $\forall x(P(x)\rightarrow\exists y\, Q(x,y)) \equiv \forall x\exists y (P(x)\rightarrow Q(x,y))$
- (a) "For some x, P(x) is true and for all y Q(x,y) is true which is not implies L"
- $\exists x (P(x) \land\forall y\, Q(x,y)) = \exists x\forall y (P(x) \land Q(x,y))$
- $[\exists x\forall y (P(x) \land Q(x,y))] \rightarrow [\forall x\exists y (P(x)\rightarrow\exists y\, Q(x,y))]$ is false.
- $(P \land Q) \rightarrow(P \rightarrow Q)$ tautology but $\exists x\forall y \rightarrow \forall x\exists y$ not valid .
- (b) "For every x and every y Q(x,y) is true which implies L"
- $[\forall x\forall y\, Q(x,y)] \rightarrow [\forall x\exists y (P(x)\rightarrow Q(x,y))]$ is true irrespective of P(x).
- $Q \rightarrow (P \rightarrow Q)$ is tautology and $\forall x\forall y \rightarrow \forall x\exists y$ valid implication.
- (c) "These exists some y such that for every x if P(x) is true then Q(x,y) is also true which implies L"

- ∃y∀ x (P (x)→Q (x,y))) → ∀x∃y (P(x)→Q (x,y)) is true .
- ∃y∀x → ∀x∃y is valid implications
- While ∀x∃y → ∃y∀x is not valid .
- (d) "There exists some x for which P(x) is true and also for some y Q(x,y) is true which cannot implies L"
- ∃x∃y (P (x) ∧ Q (x,y)) → ∀x∃y (P(x)→Q (x,y)) is false.
- (P ∧ Q) →(P →Q) is tautology but ∃x∃y → ∀x∃y is not valid
- Ans : (b),(c)

- **GATE CS 2024 | Set 2 | Question: 2**
- Let p and q be the following propositions:
- p : Fail grade can be given. q : Student scores more than 50% marks.
- Consider the statement: "*Fail grade cannot be given when student scores more than 50% marks.*"
- Which one of the following is the CORRECT representation of the above statement in propositional logic?
- A. q→¬p B. q→p C. p→q D. ¬p → q
- A variety of terminology is used to express *p* → *q*.
- "if *p*, then *q*","if *p*, *q*" , "p only if q" , "*q* if *p*", "*q* when *p*" ,"*q* unless ¬*p*","p implies q
- Q when P means P→Q.
- ¬P : Fail grade cannot be given
- Fail grade cannot be given when student score more than 50% marks.
- Q→¬P
- Ans : A. q→¬p

- ***GATE DA 2024 | Question: 19**
- Let x and y be two propositions. Which of the following statements **is a tautology /are tautologies**?
- (A) $(\neg x \land y) \Rightarrow (y \Rightarrow x)$ (B) $(x \land \neg y) \Rightarrow (\neg x \Rightarrow y)$
- (C) $(\neg x \land y) \Rightarrow (\neg x \Rightarrow y)$ (D) $(x \land \neg y) \Rightarrow (y \Rightarrow x)$
- If consequence is false and hypothesis is true, then we will get False in the truth table.
- (A) $(\neg x \land y) \Rightarrow (y \Rightarrow x)$,Let y=T and x=F
- Then (T ∧T) ⟹(T ⟹F) = T ⟹F=F Not tautology
- (B) $(x \land \neg y) \Rightarrow (\neg x \Rightarrow y)$,Let x=F and y=F
- Then (F∧T) ⟹(T ⟹F) =F ⟹F=T tautology
- (C) $(\neg x \land y) \Rightarrow (\neg x \Rightarrow y)$, Let x=F and y=F
- Then (T∧F) ⟹(T ⟹F) =F ⟹F=T tautology
- (D) $(x \land \neg y) \Rightarrow (y \Rightarrow x)$, Let x=F and y=T
- Then (F∧F) ⟹(T ⟹F) =F ⟹F=T tautology
- Ans : (B),(C),(D)

- ***GATE DA 2024 | Question: 44**
- Let *game(ball, rugby)* be true if the ball is used in rugby and false otherwise.
- Let *shape(ball, round)* be true if the ball is round and false otherwise.
- Consider the following logical sentences:
- s_1: $\forall ball\ \neg\ game(ball, rugby) \Rightarrow shape(ball, round)$
- s_2: $\forall ball\ \neg\ shape(ball, round) \Rightarrow game(ball, rugby)$
- s_3: $\forall ball\ game(ball, rugby) \Rightarrow \neg\ shape(ball, round)$
- s_4: $\forall ball\ shape(ball, round) \Rightarrow \neg\ game(ball, rugby)$
- Which of the following choices is/are logical representations of the assertion,
- "All balls are round except balls used in rugby"?
- (A) $s_1 \wedge s_3$ (B) $s_1 \wedge s_2$ (C) $s_2 \wedge s_3$ (D) $s_3 \wedge s_4$
- s_1: All ball which are not played in rugby game are round shape
- s_2: All ball which are not round shape are played in rugby game
- s_3: All ball played in rugby game are not round shape
- s_4: All round shape ball are not played in rugby game
- (A) $s_1 \wedge s_3$ = the rugby ball are not round in shape that all balls except the rugby ball are round.
- (B) $s_1 \wedge s_2$ = rugby balls are not round and if the shape of the ball is not round it rugby ball but the conjunction doesn't provide the above statement.
- (C) $s_2 \wedge s_3$ = all rugby ball are not in round shape ,same
- (D) $s_3 \wedge s_4$ = not same as statement
- Ans : (A),(C)