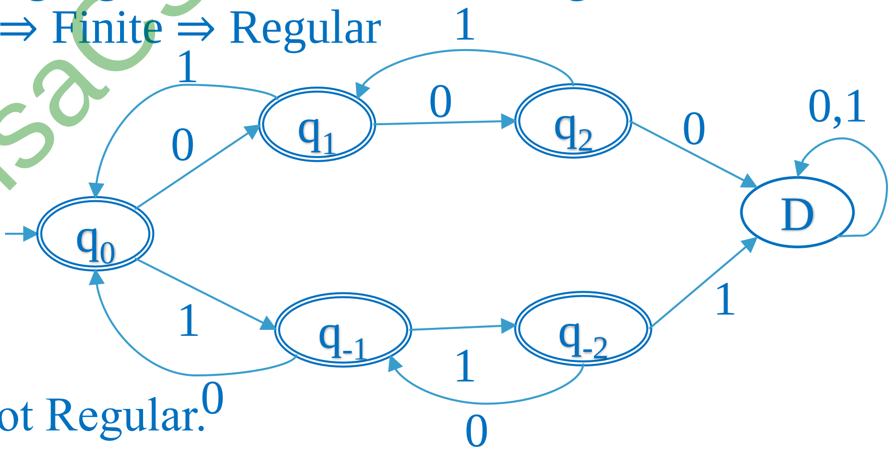
Theory of Computation
Chapter 1:Regular Language

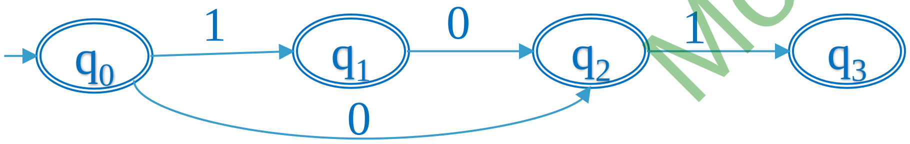
GATE CS Previous Questions Chapter wise Solved By *Monalisa Pradhan*

- GATE CS 2006,Q29: If s is a string over (0 + 1)* then let $n_0(s)$ denote the number of 0's in s and $n_1(s)$ the number of 1's in s. Which one of the following languages is not regular?
- (A) $L=\{s \in (0+1)^* \mid n_0(s)$ is a 3 digit prime$\}$
- (B) $L=\{s \in (0+1)^* \mid$ for every prefix s' of s, $|n_0(s') - n_1(s')| \leq 2\}$
- (C) $L=\{s \in (0+1)^* \mid |n_0(s) - n_1(s)| \leq 4\}$
- (D) $L=\{s \in (0+1)^* \mid n_0(s) \bmod 7 = n_1(s) \bmod 5 = 0\}$
- (A) Since 3-digit prime numbers are finite so language is finite, hence it is regular.
- $n_0(s) = \{101, 103, 107, \ldots. 977, 983, 991, 997\} \Rightarrow$ Finite $\Rightarrow$ Regular
- (B) $n_0(s') - n_1(s') \leq -2,-1,0,1,2$
- $\{\epsilon, 0,1,00,11,010,100,101,1101,1000\ldots.\}$
- Not accept $\{000^+, 111^+, 10000, \ldots\}$
- The language is Regular
- (C) infinite comparisons between 0's and 1's.
- $n_0(s) - n_1(s) \leq -4,-3,-2,-1,0,1,2,3,4$
- We need a stack for comparison so it's a CFL not Regular.
- (D) Same as number of '0' divisible by 7 and number of '1' divisible by 5.
- It's a compound automata .We can design DFA for it in 35 states.
- Hence Regular.
- Ans : (C) $L=\{s \in (0+1)^* \mid |n_0(s) - n_1(s)| \leq 4\}$



- GATE CS 2010,Q39(2Mark): Let L = {w ∈ (0 + 1)* | w has even number of 1s}, i.e. L is the set of all bit strings with even number of 1s. Which one of the regular expression below represents L?
- **(A)** (0*10*1)*
  (B) 0*(10*10*)*
  (C) 0*(10*1*)*0*
  (D) 0*1(10*1)*10*
- The best way to find correct answer is option elimination method.
- We will guess strings which has even number of 1's and that is not generated by wrong options OR which generate strings which doesn't have even number of 1's.
- Option A: doesn't generate string such as { 110, 1100,....}
- Option C: generate string such as {1, 111,....} which have odd number of 1's.
- Option D: doesn't generate strings such as { ∈,11101, 1111101, ….}.
- **Ans :(B)** 0*(10*10*)*

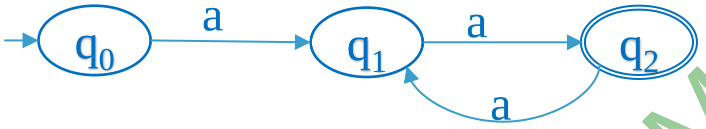
- GATE CS 2010,Q41(2 Mark):Let w be any string of length n is {0, 1}*. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automaton that accepts L?
- (A) n-1 (B) n (C) n+1 (D) 2^{n-1}
- In order to accept any string of length "n" with alphabet {0,1}, we require an NFA with "n+1" states.
- Let a strings of length "3" such as "101", the NFA require 4 states
- L is set of all substrings of 'w', if w = '101', then L={ ϵ, 0, 1, 10, 01, 101}.



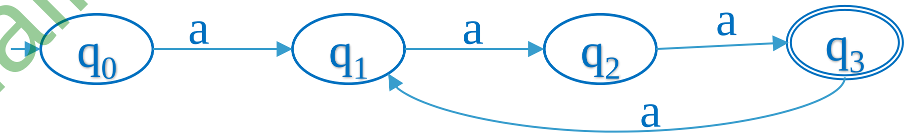
- For |w|=3 we require 4 states.
- For n length string, n+1 states are required
- Ans (C) n+1

- GATE CS 2011,Q8(1 Mark):Which of the following pairs have DIFFERENT expressive power?
  (A) Deterministic finite automata(DFA) and Non-deterministic finite automata(NFA)
  (B) Deterministic push down automata(DPDA)and Non-deterministic push down automata(NPDA)
  (C) Deterministic single-tape Turing machine and Non-deterministic single-tape Turing machine
  (D) Single-tape Turing machine and multi-tape Turing machine
- (A) E(DFA)=E(NFA)
- (B) E(DPDA)≠E(NPDA) NPDA is more powerful than DPDA.
- (C) E(DTM)=E(NTM)
- (D) E(Single-tape Turing machine)=E(multi-tape Turing machine)
- Ans: (B)

- GATE CS 2011,Q42(2 Mark):Definition of a language L with alphabet $\{a\}$ is given as following . L={a^{nk}|k>0, and n is a positive integer constant} What is the minimum number of states needed in DFA to recognize L?
- **(A)** k+1 **(B)** n+1 **(C)** $2^{(n+1)}$ **(D)** $2^{(k+1)}$
- Given that n is a constant.
- lets n = 2,
- L = a^{2k}, k>0
- L accept even no. of a's except 'ε'.
- L={aa ,aaaa ,aaaaaa,……..}

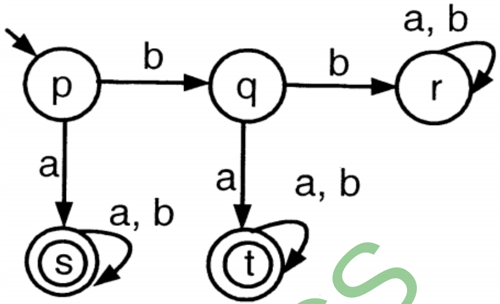


- lets n = 3,
- L = a^{3k}, k>0
- L={aaa ,aaaaaa ,aaaaaaaaa,……..}



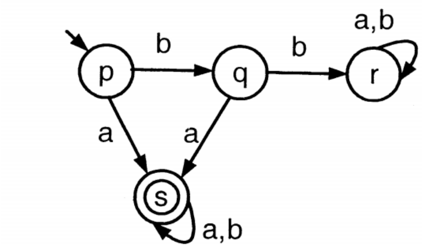
- Number of states required for n=2 is 2+1 = 3.
- Number of states required for n=3 is 3+1 = 4.
- So for a^{nk}, (n+1) states will be required.
- **Ans :(B)** n+1

- GATE CS 2011,Q45(2 Mark):A deterministic finite automation (DFA)D with alphabet {a,b} is given below.

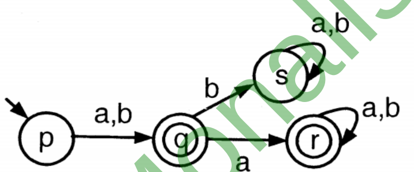


- Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

(A)

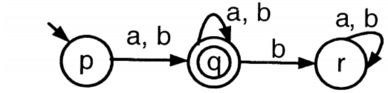


(B)

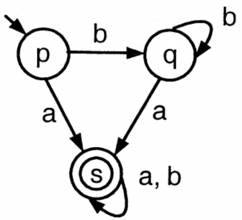


q	X			
r	X	X		
s	X	X	X	
t	X	X	X	=
	p	q	r	s

(C)

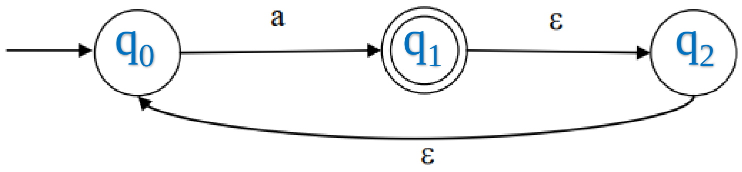


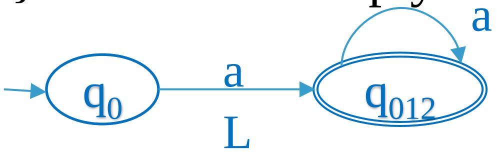
(D)

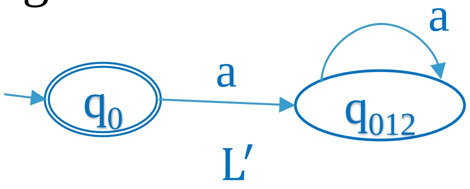


- **Ans: (A)**

- GATE CS 2012,Q12(1 Mark):What is the complement of the language accepted by the NFA shown below ?Assume $\Sigma = \{a\}$ and ε is the empty string.





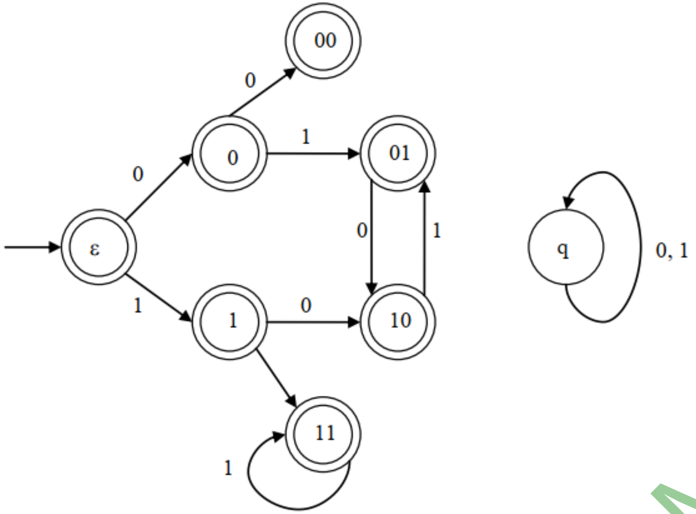


(A) ∅ (B) {ε} (C) a^* (D) {a , ε}

- The $\Sigma = \{a\}$ and the given NFA accepts the strings {a, aa, aaa, aaaa, ……….}
- The language accepted by the NFA $\{a^+\}$
- Hence the complement of language is: $\{a^* - a^+\} = \{\epsilon\}$
- Ans :**(B)** $\{\epsilon\}$

- GATE CS 2012,Q25(1 Mark):Given the language $L = \{ab, aa, baa\}$, which of the following strings are in L^*?
  1) *abaabaaabaa*
  2) *aaaabaaaa*
  3) *baaaaabaaaab*
  4) *baaaabaa*
  (A) 1, 2 and 3 (B) 2, 3 and 4
  (C) 1, 2 and 4 (D) 1, 3 and 4
- L* will contain all those strings which can be obtained by any combination (and repetition) of the strings in language i,e, from L= {ab, aa, baa}
- String 1: abaabaaabaa : ab aa baa ab aa
- String 2: aaaabaaaa : aa aa baa aa
- String 3: baaaaabaaaab: baa aa ab aa aa b, because of the last "b" the string cannot belong to L*.
- String 4: baaaabaa : baa aa ab aa
- Ans (C) 1, 2 and 4

- GATE CS 2012,Q46(2 Mark):Consider the set of strings on {0,1} in which, every substring of 3 symbols has at most two zeros. For example, 001110 and 011001 are in the language, but 100010 is not. All strings of length less than 3 are also in the language. A partially completed DFA that accepts this language is shown below. The missing arcs in the DFA are



(A)

	00	01	10	11	q
00	1	0			
01				1	
10	0				
11			0		

(B)

	00	01	10	11	q
00		0			1
01		1			
10				0	
11		0			

(C)

	00	01	10	11	q
00		1			0
01		1			
10			0		
11		0			

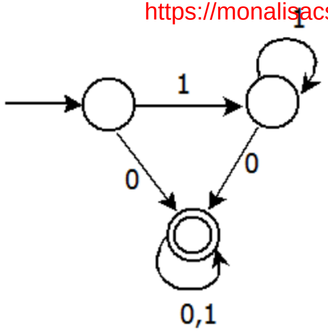
(D)

	00	01	10	11	q
00		1			0
01				1	
10	0				
11			0		

- From the state ‘00’ if another ‘0’ comes then the string is going to be rejected.
- From state ‘00’ by transition ‘0’ will go to state ‘q’. So option A and B are eliminated.
- From state ‘01’ by ‘1’ it will go to ‘11’ option C rejected.
- Ans :D

- GATE CS 2013,Q8(1 Mark):Consider the languages $L1 = \Phi$ and $L2 = \{a\}$.
- Which one of the following represents $L_1 L_2^* \cup L_1^*$?
  (A) $\{\epsilon\}$ (B) Φ (C) a^* (D) $\{\epsilon, a\}$
- $L_1 L_2^* = \Phi \cdot a^* = \Phi$
- Φ is empty language .concatenation of Φ with any other language is Φ.
- $L_1^* = \Phi^* = \epsilon$
- $L_1 L_2^* \cup L_1^* = \Phi \cup \epsilon = \epsilon$
- Ans : (A) $\{\epsilon\}$

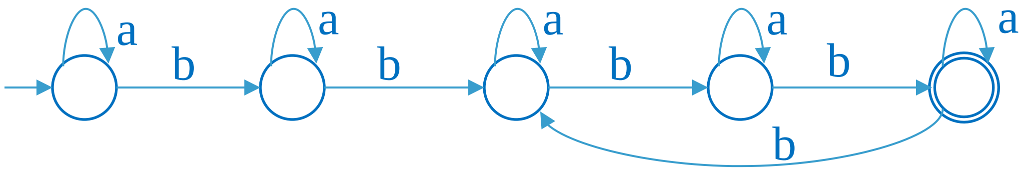
- GATE CS 2013,Q33(2 Mark):Consider the DFA A given below.



- Which of the following are **FALSE**?

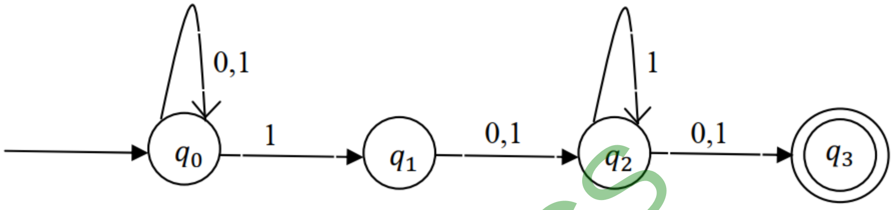
  1. Complement of *L*(*A*) is context-free.
  2. *L*(*A*) = *L*((11*0+0)(0 + 1)*0*1*)
  3. For the language accepted by *A*, *A* is the minimal DFA.
  4. *A* accepts all strings over {0, 1} of length at least 2.
- (A) 1 and 3 only (B) 2 and 4 only (C) 2 and 3 only (D) 3 and 4 only
- L(A) is regular ,its complement is also regular (by closure property)
- RL⊂ CFL . So Complement of L(A) is context-free.
- Regular expression =(11*0 +0) (0+1)*
- If we write 0*1* after this it will not have any effect, it is equivalent to (11*0 +0) (0+1)*0*1*
- Its not minimal DFA.Hence statement 3 is false.
- DFA accept string '0' ,|0|=1, so the statement 4 is false statement.
- Ans : (D) 3 and 4 only

- GATE CS 2014 Set-1,Q15(1 Mark):Which one of the following is TRUE?
- (A) The language $L=\{a^n b^n \mid n\geq 0\}$ is regular.
- (B) The language $L=\{a^n \mid n \text{ is prime}\}$ is regular.
- (C) The language $L=\{w \mid w \text{ has } 3k+1 \text{ b's for some } k\in N \text{ with } \Sigma=\{a,b\}\}$ is regular.
- (D) The language $L=\{ww \mid w\in\Sigma^* \text{ with } \Sigma=\{0,1\}\}$ is regular
- (A)The Language $L= \{a^n b^n \mid n>=0\}$ is CFL but not regular, as it requires comparison between a's and b's.
- (B)$L = \{a^n \mid n \text{ is prime}\}$ is CSL, as calculation of "n is prime" can be done by LBA
- (D)$L = \{ww \mid w \in \Sigma^*\}$ is CSL.
- (C)$L = \{ w \mid w \text{ has } 3k+1 \text{ b's for some } k \in N\}$ is regular.
- $k=\{1,2,3,...\}$,$|w|_b= \{4, 7, 10,\dots\}$ and number of a's can be anything.
- The DFA will be



- Ans :(C)

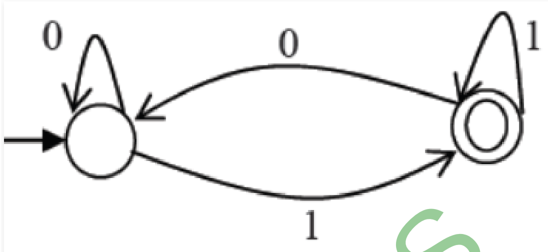
- GATE CS 2014 Set-1,Q16(1 Mark): Consider the finite automaton in the following figure. What is the set of reachable states for the input string 0011?
- (A) $\{q_0, q_1, q_2\}$
- (B) $\{q_0, q_1\}$
- (C) $\{q_0, q_1, q_2, q_3\}$
- (D) $\{q_3\}$



	0	0	1	1	
	q_0	q_0	q_0	q_0	$\delta(q_0, 0011) = q_0$
→ q_0	q_0	q_0	q_0	q_1	$\delta(q_0, 0011) = q_1$
	q_0	q_0	q_1	q_2	$\delta(q_0, 0011) = q_2$

- $\delta(q_0, 0011) = \{q_0, q_1, q_2\}$
- Ans: (A) $\{q_0, q_1, q_2\}$

- GATE CS 2014 Set-1,Q36(2 Mark):Which of the regular expressions given below represent the following DFA?

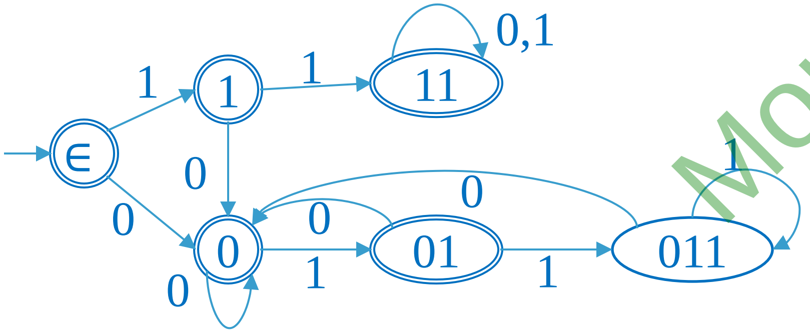


- I) 0*1(1+00*1)*
- II) 0*1*1+11*0*1
- III) (0+1)*1
- **(A)** I and II only **(B)** I and III only **(C)** II and III only **(D)** I, II, and III
- (I) and (III) represent DFA.
- (II) 0101,1011, belongs to language but regular expression doesn't accept.
- Ans :(B) I and III only

- GATE CS 2014 Set-2,Q15(1 Mark):If $L_1=\{a^n|n\geq0\}$ and $L_2=\{b^n|n\geq0\}$, consider

  (I) $L_1.L_2$ is a regular language (II) $L_1.L_2=\{a^nb^n|n\geq0\}$
- Which one of the following is CORRECT?
- (A) Only (I) (B) Only (II) (C)Both (I) and (II) (D)Neither (I) nor (II)
- The regular expression equivalent to L_1 and L_2 are a* and b* respectively.
- Since L_1 and L_2 both are regular languages and regular languages are closed under concatenation. So their concatenation (i.e., $L_1 \cdot L_2$) must also be a regular language.
- (I) $L_1 \cdot L_2$ =a*b* or $\{ a^mb^n \mid m,n \geq 0\}$ so Regular.
- (II) $L_1.L_2=\{ a^nb^n \mid n \geq 0\}$ is CFL
- Hence, statement (I) is True but statement (II) is False.
- Ans : (A) Only (I)

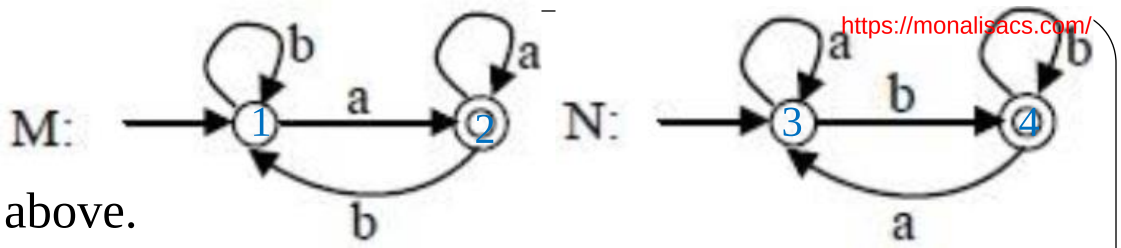
- GATE CS 2014 Set-2,Q36(2 Mark): Let $L_1 = \{w \in \{0,1\}^* \mid w$ has at least as many occurrences of (110)'s as (011)'s$\}$. Let $L_2 = \{w \in \{0,1\}^* \mid w$ has at least as many occurrences of (000)'s as (111)'s$\}$. Which one of the following is TRUE?
- **(A)** L1 is regular but not L2 **(B)** L2 is regular but not L1
- **(C)** Both L2 and L1 are regular **(D)** Neither L1 nor L2 are regular
- In L_1 any string must satisfy the condition: {Number of occurrences of (110)} ≥ {Number of occurrences of (011)}
- $L_1 = \{\in, 0, 1, 00, 01, 10, 11, 000, 001, 010, 100, 101, 110, 111, \ldots, 11011, 1100110, 0^*, 1^*, 11(0+1)^*\}$
- $L_1 \neq \{011, 0011, 011011 \ldots\}$
- We cannot have two 110′s in a string without a 011 or vice verse



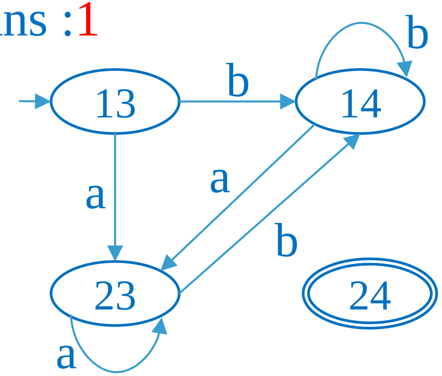
- But language L_2 requires infinite comparison to count the occurrences of (000's) and (111's), hence it is not regular.
- Ans : **(A)** L1 is regular but not L2

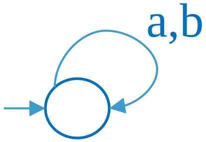
- GATE CS 2014 Set-3,Q15,1 Mark:The length of the shortest string NOT in the language over Σ = {a, b} of the following regular is expression is ________________.
- a*b*(ba)*a*
- Length 0={ϵ}
- Length 1={a,b}
- Length 2={aa,ab,ba,bb}
- Length 3={aaa,aab,aba,abb,baa,bab,bba,bbb}
- It doesn’t generate the string “bab”, hence the shortest string not generated by regular expression has length 3 (string “bab”).
- Ans: 3

- GATE CS 2014 Set-3,Q16,1 Mark:Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ*. Which one of the following is **TRUE**?
- (A) Both 2^{Σ^*} and Σ* are countable
- (B) 2^{Σ^*} is countable and Σ* is uncountable
- (C) 2^{Σ^*} is uncountable and Σ* is countable
- (D) Both 2^{Σ^*} and Σ* are uncountable
- Let ∑ ={a, b} then ∑* = { ε, a, b, aa, ba, bb, ……………….}
- "Set of all strings over any finite alphabet are Countable". ∑* is countable.
- ∑* is countably infinite But 2^{Σ^*} is Uncountable, which can be proved using Diagonalization Method. This theorem says- "If ∑* is countably infinite then 2^{Σ^*} is Uncountable".
- Ans:(C) 2^{Σ^*} is uncountable and Σ* is countable

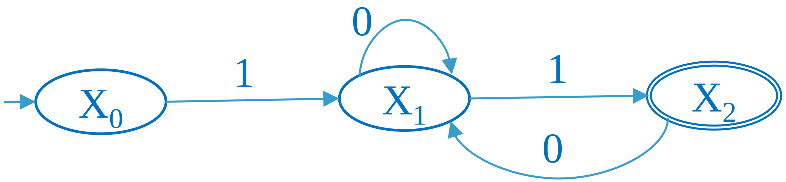


- GATE CS 2015 Set-1,Q52,2 Mark:
- Consider the DFAs M and N given above.
  The number of states in a minimal DFA that accepts the language L(M) ∩ L(N) is ____.
- L(M)={all strings end with 'a'}
- L(N) ={all strings end with 'b'}
- L(M)={a,aa,ba,aaa,aba,baa,bba,……}
- L(N)={b,ab,bb,aab,abb,bab,bbb,……}
- L(M)∩ L(N)=ϕ
- For an empty language, only one state is required in DFA.
- Ans :1





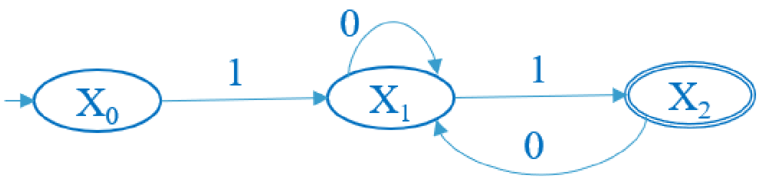
- GATE CS 2015 Set-2,Q35,2 Mark: Consider alphabet $\sum = \{0, 1\}$, the null/empty string λ and the sets of strings X_0, X_1 and X_2 generated by the corresponding non-terminals of a regular grammar. X_0, X_1 and X_2 are related as follows:
- $X_0 = 1X_1$
- $X_1 = 0X_1 + 1 X_2$
- $X_2 = 0 X_1 + \{\lambda\}$
- $X_0 = 1X_1$
- $= 11X_2$
- $=11\in =11$
- Which one of the following choices precisely represents the strings in X_0?
- (A) 10 (0* + (10)*)1
- (B) 10 (0* + (10)*)*1
- (C) 1(0* + 10)*1
- (D) 10 (0 + 10)*1 + 110 (0 + 10)*1
- This is a Right linear grammar. Start symbol is the initial state.∈ is in final state.
- Number of variable =number of state.
- Convert the given Grammar to a state diagram.
- We are asked to find the set of strings generated by X_0
- $X_0 = 1(0+10)^* 1$
- $X_1 = (0+10)^*1$
- $X_2 = 0(0+10)^*1+ \in$
- Ans : (C) 1(0* + 10)*1



- Arden's Theorem
- $R=Q+RP \Rightarrow R=QP^*$
- If $R=Q+PR$
- $=Q+P(Q+PR)=Q+PQ+P^2R$
- $= Q+PQ+P^2Q+P^3Q\ldots\ldots=(\in+P+P^2+P^3\ldots\ldots)Q=P^*Q$
- If $R=Q+PR \Rightarrow R= P^*Q$
- $X_0 = 1X_1$, $X_1 = 0X_1+ 1\ X_2$, $X_2 = 0\ X_1 + \in$
- $X_1 = 0X_1+1(0\ X_1 + \in)$
- $=0X_1+10X_1+1$
- $=(0+10)\ X_1+1$
- $=(0+10)^*1$
- $X_2 =0(0+10)^*1+ \in$
- $X_0 =1(0+10)^*1$

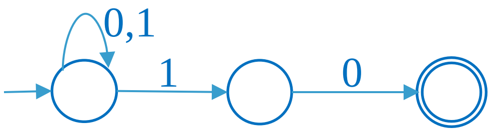
$$X_0 = 1X_1$$
$$X_1 = 0X_1 + 1\ X_2$$
$$X_2 = 0\ X_1 + \{\lambda\}$$

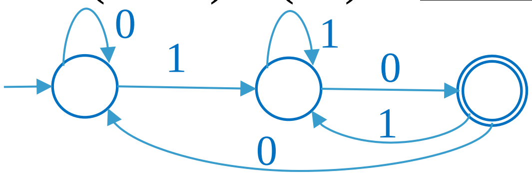
- $X_0 = 1(0+10)^*\ 1$
- $X_1 = (0+10)^*1$
- $X_2 = 0(0+10)^*1+ \in$
- Ans : (C) $1(0^* + 10)^*1$



- GATE CS 2015 Set-2,Q51,2 Mark:Which of the following languages is/are regular?
- L_1: $\{wxw^R \mid w, x \in \{a, b\}^* \text{ and } |w|, |x| > 0\}$ w^R is the reverse of string w
- L_2: $\{a^n b^m \mid m \neq n \text{ and } m, n \geq 0\}$
- L_3: $\{a^p b^q c^r \mid p, q, r \geq 0\}$
- **(A)** L_1 and L_3 only **(B)** L_2 only **(C)** L_2 and L_3 only **(D)** L_3 only
- L_1: All strings of length 3 or more, start and end with same symbol, as everything in middle is consumed by x as per the definition.
- L_2: In this number of a's is dependent on number of b's. So PDA is needed.
- L_3: Any number of a's followed by any number of b's followed by any number of c's. Hence Regular.
- Ans: L_1 and L_3 only

- GATE CS 2015 Set-2,Q53,2 Mark: The number of states in the minimal deterministic finite automaton corresponding to the regular expression (0 + 1) * (10) is ________ .



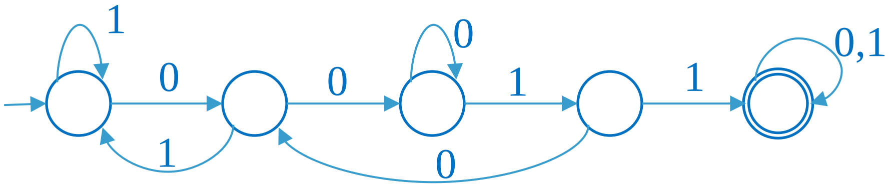


- L={10,010,110,0010,0110,1010,1110,10110,…..}
- L={Every string end with 10}
- Number of states in minimal DFA is 3.
- If in L every string ends with substring s or suffix s i.e w=xs, |s|=n.
- Then number of state required in minimal DFA=n+1.
- |10|=2
- # state=2+1=3
- Ans :3

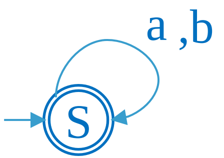
- GATE CS 2015 Set-3,Q18,1 Mark: Let T be the language represented by the regular expression Σ*0011Σ* where Σ = {0, 1}. What is the minimum number of states in a DFA that recognizes L' (complement of L)?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 8

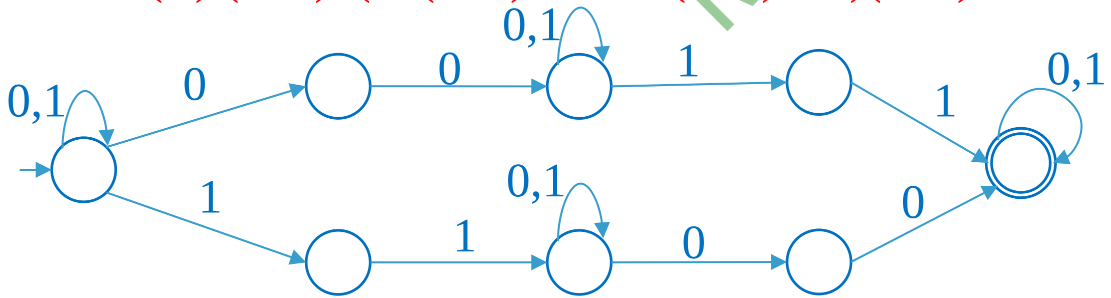
- If Language contain substring s, i.e w=xsx , |s|=n
- Then number of state required in Minimal DFA = n+1
- Regular expression Σ*0011Σ*.
- L={Every string contain substring '0011'}
- |0011|=4 ,So 4+1=5 State
- Complement have same number of state. Only final state change to non final and non final state change to final state.
- Ans :(B)5



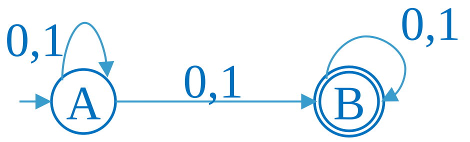
- GATE CS 2016 Set-1,Q16,1 Mark: Which of the following languages is generated by the given grammar? $S \rightarrow aS|bS|\varepsilon$
- (A) $\{a^n b^m \mid n,m \geq 0\}$
- (B) $\{w \in \{a,b\}^* \mid w$ has equal number of a's and b's$\}$
- (C) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$
- (D) $\{a,b\}^*$
- $L=\{\varepsilon,a,b,aa,ab,ba,bb,aaa,aba,abaabab,\ldots..\}$
- Ans: (D) $\{a,b\}^*$
- Draw a DFA using given grammar.

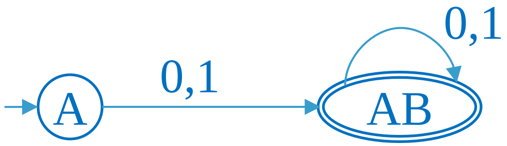


- GATE CS 2016 Set-1,Q18,1 Mark: Which one of the following regular expressions represents the language: *the set of all binary strings having two consecutive* 0s *and two consecutive* 1s?
  (A) (0+1)*0011(0+1)* + (0+1)*1100(0+1)*
  (B) (0+1)*(00(0+1)*11+11(0+1)*00)(0+1)*
  (C) (0+1)*00(0+1)* + (0+1)*11(0+1)*
  (D) 00(0+1)*11+11(0+1)*00
- L={0011,1100,00011,00110,001011,11010100,1001011……}
- A:Set of strings which either have 0011 or 1100 as substring. Doesn't generate ‘001011’
- C: Set of strings which either have 00 or 11 as substring. Generates string ’00’ which doesn’t have two consecutive 1’s.
- D: Set of strings which start with 11 and end with 00 or start with 00 and end with 11. doesn’t generate string ‘00110’
- Ans : (B) (0+1)*(00(0+1)*11+11(0+1)*00)(0+1)*



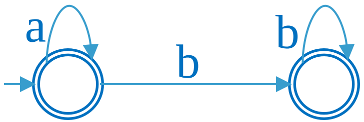
- GATE CS 2016 Set-2,Q16,1 Mark: The number of states in the minimum sized DFA that accepts the language defined by the regular expression (0+1)*(0+1)(0+1)* is ________
- The regular expression generates the min string ‘0’ or ‘1’ .
- So, the DFA has two states.
- Ans: 2





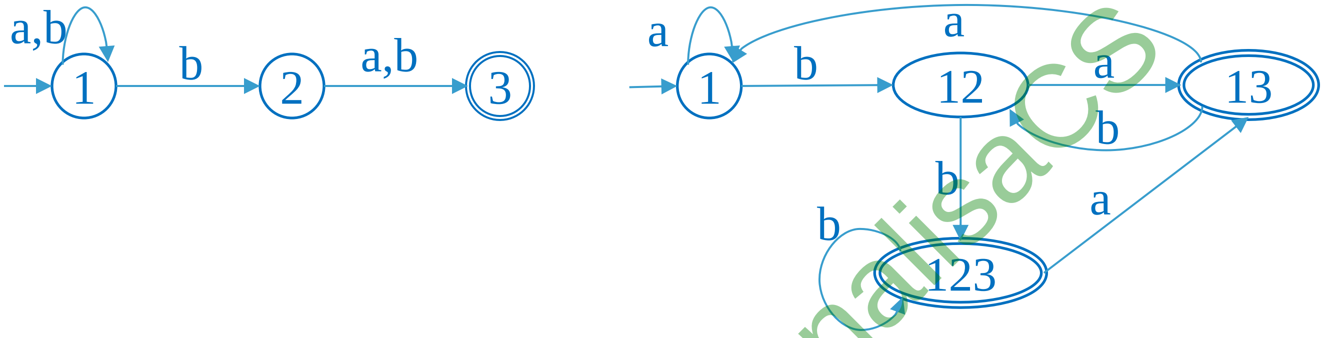
- GATE CS 2016 Set-2,Q17,1 Mark:
- Language L_1 is defined by the grammar: $S_1 \rightarrow aS_1b|\varepsilon$
- Language L_2 is defined by the grammar: $S_2 \rightarrow abS_2|\varepsilon$
- Consider the following statements:
- P: L_1 is regular Q: L_2 is regular
- Which one of the following is **TRUE**?
- **(A)** Both P and Q are true **(B)** P is true and Q is false
  (C) P is false and Q is true **(D)** Both P and Q are false
- L_1 ={ε,ab,aabb,aaabbb….}
- $L_1 = \{a^n b^n \mid n \geq 0\}$ CFL not RL.
- L_2 ={ε,ab,abab,ababab,….}
- $L_2 = (ab)^*$ RL
- Ans : **(C)** P is false and Q is true

- GATE CS 2016 Set-2, Q42,2 Mark:Consider the following two statements:
- **I.**If all states of an NFA are accepting states then the language accepted by the NFA is Σ*.
- **II.**There exists a regular language A such that for all languages B, A∩B is regular.
- Which one of the following is **CORRECT**?
- **(A)** Only I is true
- **(B)** Only II is true
- **(C)** Both I and II are true
- **(D)** Both I and II are false
- I is false: NFA is not a complete system , so even though all states are final state in NFA, the NFA will reject some strings.



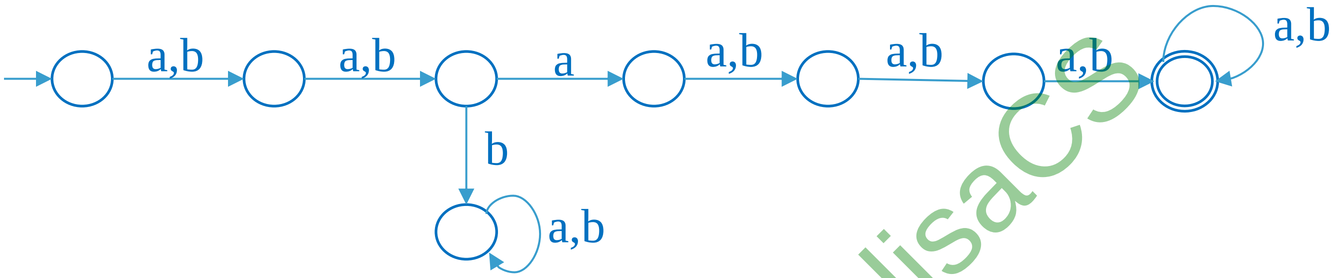
- For ex: Consider L = a*b*
- It doesn't accept string {ba,aba,abaab,babaa,…}.Hence its language can't be ∑*.
- II is true: A= ϕ is an Empty language and also RL.
- ϕ ∩B= ϕ for all language B
- Ans : **(B)** Only II is true

- GATE CS 2017 Set-1,Q22,1 Mark:Consider the language *L* given by the regular expression (a+b)*b(a+b) over the alphabet {a,b}. The smallest number of states needed in deterministic finite-state automation (DFA) accepting *L* is __________.
- 1.The NFA for regular expression: (a+b)*b(a+b)



- After converting the NFA into DFA we got 4 state
- Ans: 4
- L={2^{nd} alphabet from RHS is b}
- The minimal DFA that accept n^{th} symbol from RHS is fixed contain 2^n state,Number of final state= 2^{n-1}
- $2^n = 2^2 = 4$

- GATE CS 2017 Set-2,Q25,1 Mark: The minimum possible number of a deterministic finite automation that accepts the regular language $L = \{w_1 a w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is ___________.
- $L = \{w_1 a w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = 2, |w_2| \geq 3\}$

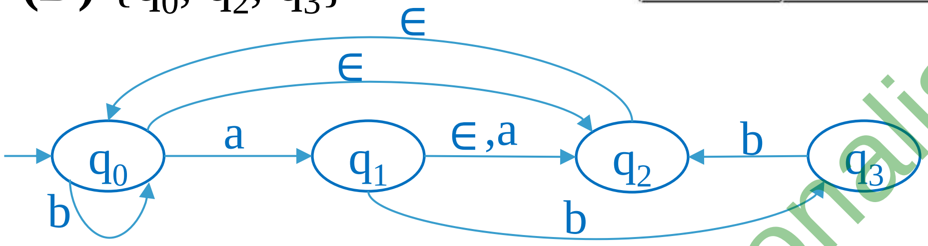


- Ans:8
- $|w_1| = 2$ need 3 state
- $|a|=1$ need 2 state
- $|w_2| \geq 3$ need 4 state
- Total=3+2+4-2+1=9-2+1=8

- GATE CS 2017 Set-2,Q39,2 Mark: Let δ denote the transition function and $\bar{\delta}$ denoted the extended transition function of the ε-NFA whose transition table is given below: Then, $\bar{\delta}$ (q_2, aba) is

δ	ϵ	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	Ø	Ø
q_3	Ø	Ø	$\{q_2\}$

- **(A)** Ø
  (B) $\{q_1, q_2, q_3\}$
  (C) $\{q_0, q_1, q_2\}$
  (D) $\{q_0, q_2, q_3\}$



	∈*	a	∈*	b	∈*	a	∈*
q_2	q_0	q_1	q_0	q_0	q_0	q_1	q_0
	q_2	ϕ	q_1	q_3	q_2	ϕ	q_1
			q_2	ϕ	q_3	ϕ	q_2

- $\in^*(q_0)=\{q_0, q_2\}$
- $\in^*(q_1)=\{q_0, q_1, q_2\}$
- $\in^*(q_2)=\{q_0, q_2\}$
- $\in^*(q_3)=\{q_3\}$

- Ans: **(C)** $\{q_0, q_1, q_2\}$

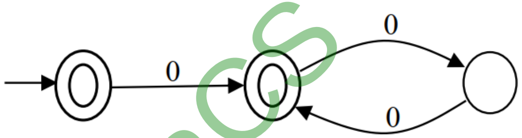
- GATE CS 2018, Q6, 1 Mark: Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?
  (A) $k \geq 2^n$ (B) $k \geq n$ (C) $k \leq n^2$ (D) $k \leq 2^n$
- # states in DFA $\leq 2^{\text{# states in NFA}}$.
- # states in NFA is ‘n’ , # states in DFA is ‘k’
- Then DFA corresponding to NFA have at most 2^n states.
- $k \leq 2^n$
- Ans : (D) $k \leq 2^n$

- GATE CS 2018,Q52,2 Mark: Given a language L, define L^i as follows:

  $L^0 = \{\varepsilon\}$

  $L^i = L^{i-1} \cdot L$ for all $i>0$

- The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$. Consider the language L_1 (over alphabet 0) accepted by the following automaton.



- The order of L_1 is ______.
- Regular expression $L_1 = \epsilon + 0\ (00)^*$
- $L_1^0 = \epsilon$
- $L_1^1 = \epsilon\ .\ (\epsilon + 0\ (00)^*) = \epsilon + 0\ (00)^* = L_1$
- $L_1^2 = L_1^1 . L_1 = (\epsilon + 0\ (00)^*)\ (\epsilon + 0\ (00)^*)$
- $= \epsilon + 0\ (00)^* + 0(00)^* + 0(00)^*0(00)^*$
- $= \epsilon + 0\ (00)^* + 0(00)^*0(00)^* = 0^*$ [ϵ + odd # zero + even #zero]
- $L_1^3 = L_1^2 . L_1 = 0^*\ (\epsilon + 0\ (00)^*)$
- $= 0^* + 0^*0(00)^* = 0^*$
- Hence $L_1^2 = L_1^3$ Or $L_1^2 = L_1^{2+1}$
- The smallest k value is 2.
- Ans :2

- GATE CS 2019,Q7,1 Mark:If L is a regular language over $\Sigma = \{a,b\}$, which one of the following languages is NOT regular?
- **(A)** $L \cdot L^R$ $\{xy \mid x \in L, y^R \in L\}$
- **(B)** $\{ww^R \mid w \in L\}$
- **(C)** Prefix (L) = $\{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{ such that } xy \in L\}$
- **(D)** Suffix (L) = $\{y \in \Sigma^* \mid \exists x \in \Sigma^* \text{ such that } xy \in L\}$
- Regular languages closed under reversal, concatenation, prefix(L), and suffix(L) property. So, languages given in option (A), (C), and (D) are regular.
- But language $L = \{ww^R \mid w \in L\}$ is infinite and not regular because it involves string matching, so it require stack. Hence, it is context-free but not regular.
- Ans:**(B)** $\{ww^R \mid w \in L\}$

- GATE CS 2019,Q15,1 Mark: For $\Sigma = \{a,b\}$, let us consider the regular language $L = \{x | x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L?
- **(A)** 3 **(B)** 5 **(C)** 9 **(D)** 24
- **Pumping Lemma for Regular Languages**:
- If L is a RL(Infinite).There exists a constant n s.t for every string w∈L,|w|≥n.
- We can break w into 3 strings ,w=xyz
- 1.$|y| \neq 0$ 2.$|xy| \leq n$ 3.$xy^kz \in L, \forall k \geq 0$
- Pumping length for a regular language makes sure that any string in that language with the length ≥pumping length has some repetition. n is the pumping length.
- $L_1 = a^{2+3k}$ $= \{a^2, a^5, a^8, a^{11}, \ldots\ldots\}$ Pumping Length=2
- $L_2 = b^{10+12k}$ $= \{b^{10}, b^{22}, b^{34}, b^{46}, \ldots\}$ Pumping length=10
- $L = L_1 \cup L_2 = \{a^2, a^5, a^8, b^{10}, a^{11}, a^{14}, a^{17}, a^{20}, b^{22} \ldots\}$
- Minimum pumping length ≥10 .
- So,pumping length 3, 9 and 5 is wrong.
- Ans :(D)24

- GATE CS 2019, Q48, 2 Mark: Let Σ be the set of all bijections from {1, …, 5} to {1, …, 5}, where id denotes the identity function, i.e. id(j) = j, ∀j. Let ° denote composition on functions. For a string $x = x_1x_2 \dots x_n \in \Sigma^n$, $n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ \dots \circ x_n$. Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$. The minimum number of states in any DFA accepting L is __________ .
- Bijection: A function f(A→B) is called a bijection if it is one to one & onto.
- If A & B are finite set then a Bijection from A to B is possible⇔ |A|=|B|
- If |A|=|B|=n then number of bijection possible from A to B=n!
- Identity Function: A function from A to A is called an identity function if f(x)=x,∀x∈A
- No of bijection =n!=5!=120
- The DFA for accepting L will have 5! = 120 states
- Ans: 120

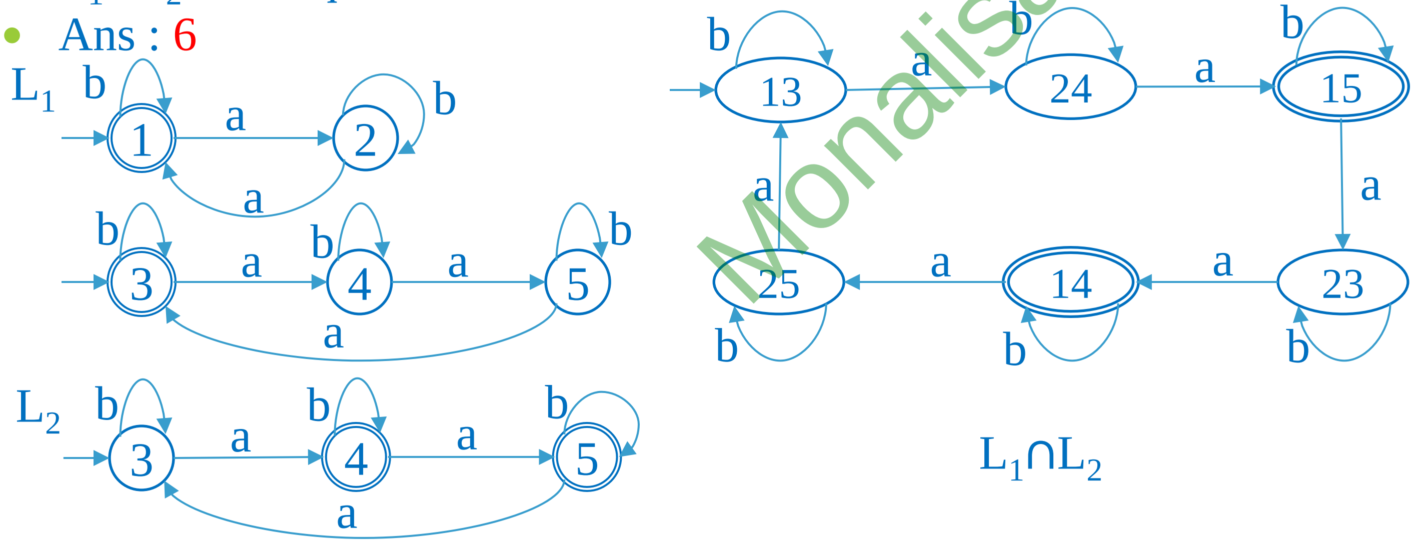
- GATE CS 2020, Q7, 1 Mark: Which one of the following regular expressions represents the set of all binary strings with an odd number of 1′s ?
  (A) ((0+1)*1(0+1)*1)*10*
  (B) (0*10*10*)*0*1
  (C) 10*(0*10*10*)*
  (D) (0*10*10*)*10*
- (A)The regular expression ((0+1)*1(0+1)*1)*10* generate string “11110” which is not having odd number of 1’s , hence wrong option.
- (B) The regular expression (0*10*10*)*0*1 always generates all string ends with ‘1’ and thus does not generate string “10” hence wrong option.
- (C) The regular expression 10*(0*10*10*)* always generate string begin with 1 and thus does not generate string ’01’ hence wrong option.
- (D) The regular expression (0*10*10*)*10* is not a generating string “01”. Hence this is also wrong
- It seems none of them is correct.
- Ans: MTA(Marks To All)

- GATE CS 2020, Q8, 1 Mark: Consider the following statements.
- **I.** If $L_1 \cup L_2$ is regular, then both L_1 and L_2 must be regular.

  II. The class of regular languages is closed under infinite union.
- Which of the above statements is/are TRUE ?

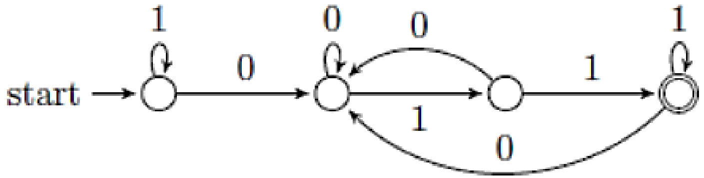
  (A) I only **(B)** II only **(C)** Both I and II **(D)** Neither I nor II
- I. $L_1=a^*b^*$ [Regular] , $L_2=a^nb^n$ [CFL]
- $L_1 \cup L_2=a^*b^*$ [Regular] hence statement 1 is wrong .
- II. $L_1=\{ab\}, L_2=\{aabb\}, L_3=\{aaabbb\}, \ldots\ldots L_{100}=\{a^{100}b^{100}\}\ldots\ldots$
- $L_1 \cup L_2 \cup \ldots\ldots\ldots L_{100} \cup \ldots\ldots=\{a^nb^n \mid n>0\}$, which is not regular.
- The class of regular language is closed under finite Union and Intersection not for infinite.
- The class of regular languages is closed under infinite union is false.
- Ans : (D) Neither I nor II

- GATE CS 2020, Q51, 2 Mark: Consider the following language.
  L = {x ∈ {a,b}* | number of a's in x is divisible by 2 but not divisible by 3}
  The minimum number of states in a DFA that accepts L is ______.
- L_1= {x∈ {a,b}* |number of a's in x is divisible by 2 } require 2 state
- ={∈,b*,aa,abab,baa,abaabba,…..}
- L_2={x∈ {a,b}* |number of a's in x is not divisible by 3} require 3 state
- ={ba,abb,aba,aabbaba,bbbaabaaa,…}
- $L_1 \cap L_2$ will require 2×3=6 state
- Ans : 6

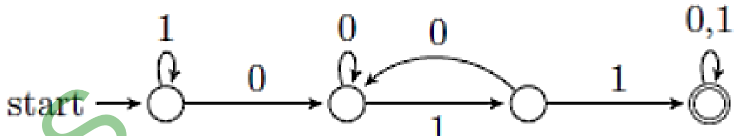


- GATE CS 2021,Set-1,Q38:Consider the following language .
- L={w∈{0,1}*|w ends with the substring 011}
- Which one of the following deterministic finite automata accepts L?

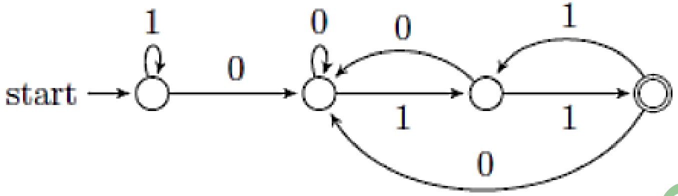
(A)



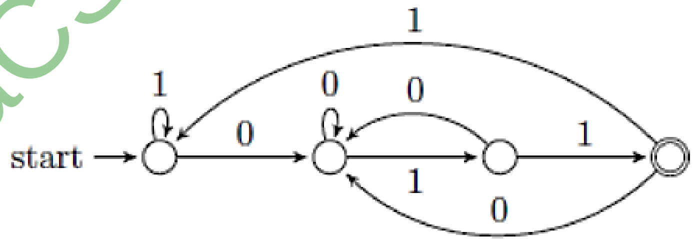
(B)

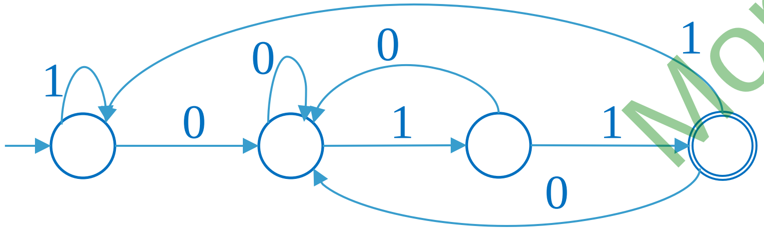


(C)



(D)

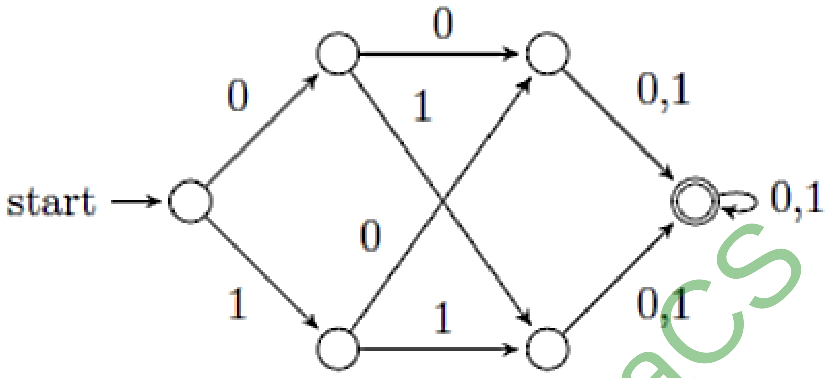




- (A) accepts 0111∉ L
- (B) accepts 0110,01111∉ L
- (C) accepts 01111 ∉ L
- (D){011,1011,011001011}∈L
- Ans :(D)

- GATE CS 2021,Set-2,Q9:Let L ⊆{0,1}* be an arbitrary regular language accepted by a minimal DFA with *k* states . Which one of the following languages must necessarily be accepted by a minimal DFA with k states?
- (A) L-{01} (B)L∪{01} (C){0,1}*-L (D)L.L
- (A,B) L-{01}, L∪{01} minimal DFA may not always by k state.
- (C) {0,1}*-L=$\overline{L}$
- L & $\overline{L}$ have equal number of state.
- Minimal DFA will have k state.
- (D) L.L more than k state.
- Ans : (C){0,1}*-L

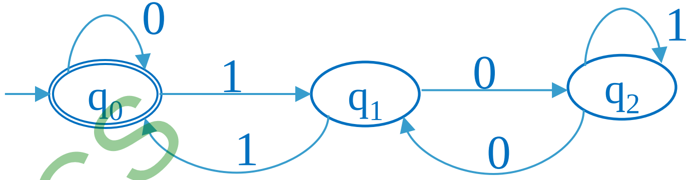
- GATE CS 2021,Set-2,Q17:Consider the following deterministic finite automaton (DFA)



- The number of strings of length 8 accepted by the above automaton is ______.
- {000,001,…111}all string of length 3
- DFA will accept all string of length 3,4,5,6,7,8,….
- L={length of string is at least 3}
- Number of string of length 8=2^8=256
- Ans: 256

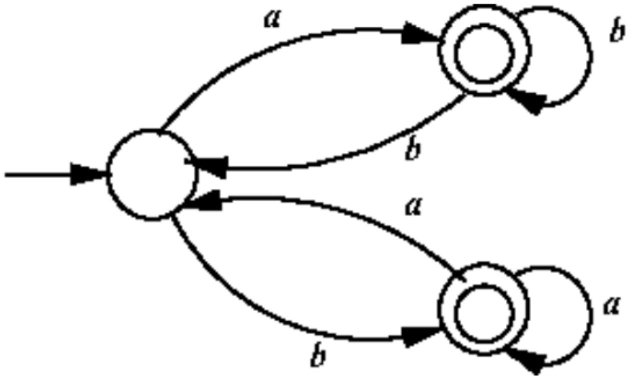
- GATE CS 2021,Set-2,Q36:Consider the following two statements about regular language:
- S_1:Every infinite regular language contains an undecidable language as a subset.
- S_2:Every finite language is regular.
- Which one of the following choices is correct?
- (A) Only S_1 is true.
- (B) Only S_2 is true.
- (C) Both S_1 and S_2 are true.
- (D) Neither S_1 nor S_2 is true.
- S_1:True
- Every infinite regular language contains Recursive enumerable Language or non recursive enumerable as subset.
- Ex: RE language on(a,b) is subset of (a+b)*
- S_2:True
- Every finite language is regular as we can design DFA for it.
- Ans: (C) Both S_1 and S_2 are true.

- GATE CS 2021,Set-2,Q47:Which of the following regular expressions represent(s) the binary numbers that are divisible by three ? Assume that the string ϵ is divisible by three.
- (A) (0+1(01*0)*1)*
- (B) (0+11+10(1+00)*01)*
- (C) (0*(1(01*0)*1)*)*
- (D) (0+11+11(1+00)*00)*



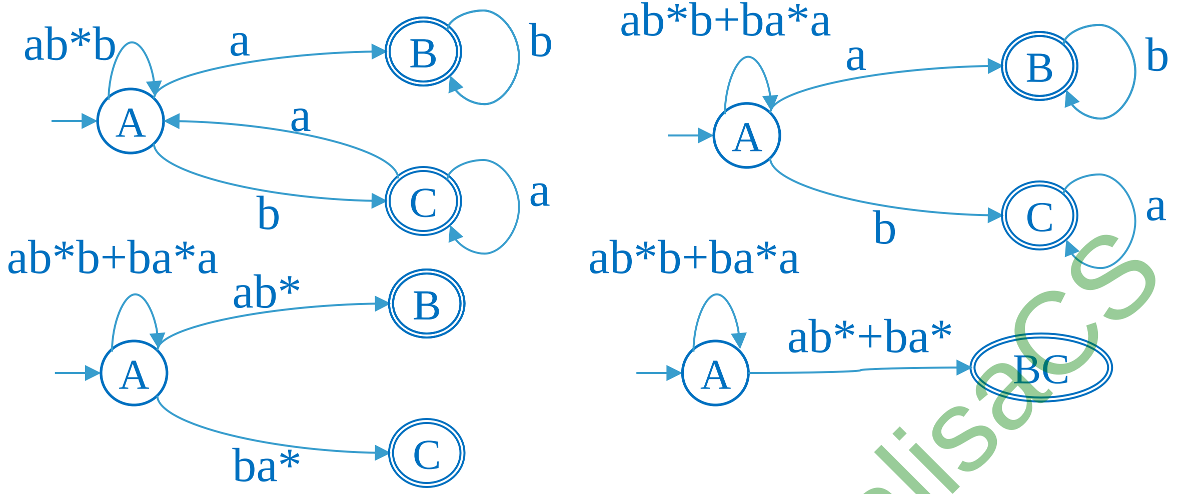
- {ϵ,0*,11,011,110,1001,1100,1111,….}
- (A) (0+1(01*0)*1)* Regular Expression
- (B) (0+11+10(1+00)*01)* Regular Expression
- (C) (0*(1(01*0)*1)*)*= (0+1(01*0)*1)* as(a+b)*=(a*b*)* Regular Expression
- (D) 11100=28 not divisible by 3
- This is not Regular Expression for above language.
- Ans : (A),(B),(C)

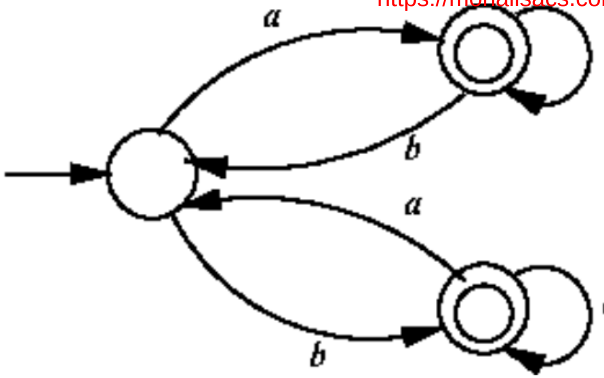
- **GATE CS 2022 | Question: 2**
- Which one of the following regular expressions correctly represents the language of the finite automaton given below?



- (A)$ab^*bab^*+ba^*aba^*$ (B)$(ab^*b)^*ab^*+(ba^*a)^*ba^*$
- (C)$(ab^*b+ba^*a)^*(a^*+b^*)$ (D)$(ba^*a+ab^*b)^*(ab^*+ba^*)$
- L={a,b,ab,ba,abb,baa,…..}
- L≠{∈,aa,bb,aaa,bbb,….}
- 1st way according to string
- (A) The RE not generate{a,b }so wrong.
- Min string={aba,bab} not accepted by FA.
- (B)This RE will not generate strings like {abbaa,baabb}
- But these are accepted by FA so wrong.
- (C) The RE will generate ∈ which is NOT accepted by the given FA .
- So wrong.
- (D)Correct.
- 2nd way run RE on NFA
- If we run all RE on FA we will found that A,B are according to individual final state .while B,C are for both final state . From that D is correct RE.

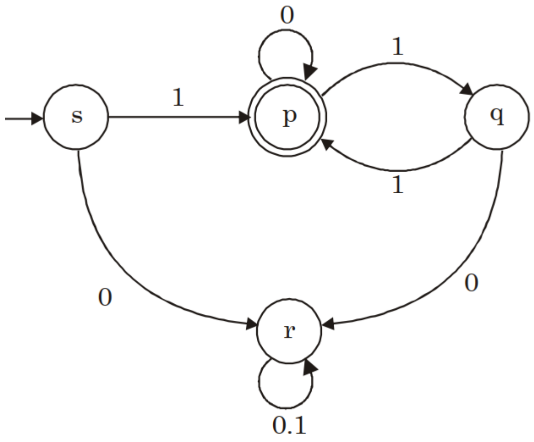
- 3rd way find RE from NFA by state elimination method



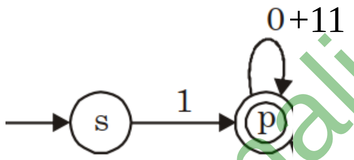


- RE=(ab*b+ba*a)*(ab*+ba*)
- Ans : (D)(ba*a+ab*b)*(ab*+ba*)

- **GATE CS 2023 | Question: 4**
- Consider the Deterministic Finite-state Automaton (DFA) A shown below. The DFA runs on the alphabet {0,1}, and has the set of states {*s*,*p*,*q*,*r*}, with s being the start state and p being the only final state.
- Which one of the following regular expressions correctly describes the language accepted by A?



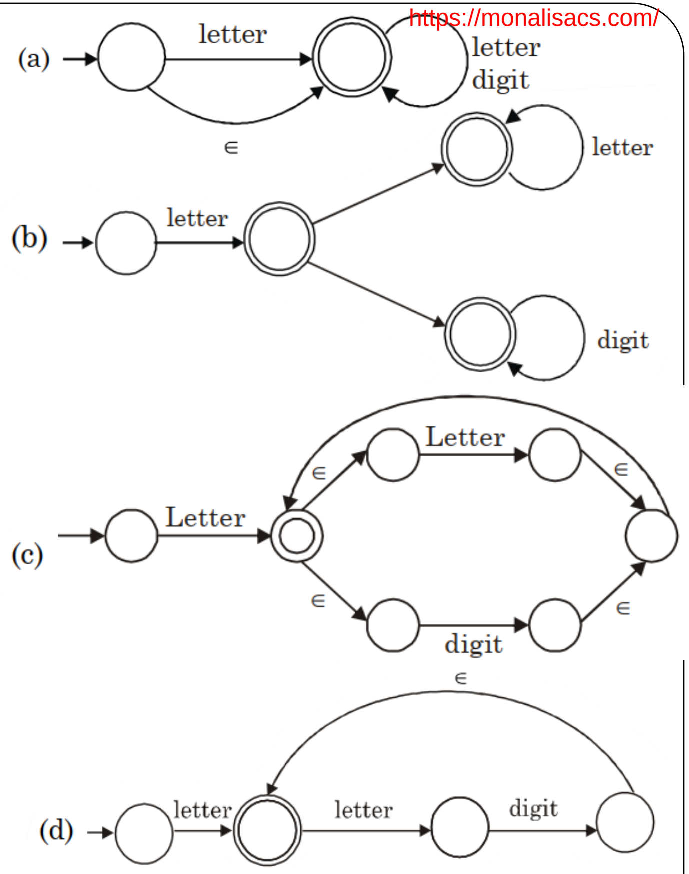
- (A)1(0*11)* (B)0(0+1)* (C)1(0+11)* (D)1(110*)*
- After removal of r
- After removal of q



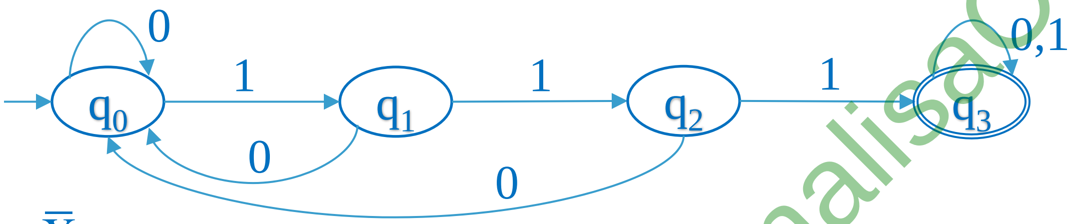
- Regular expression=1(0+11)*
- Arden's Theorem R=Q+RP ⇒R=QP*
- s=∈ , q=p1
- p=s1+q1+p0=1+p11+p0=1+p(11+0)=1(11+0)*
- Ans : (C)1(0+11)*

GATE CS 2023 | Question: 9

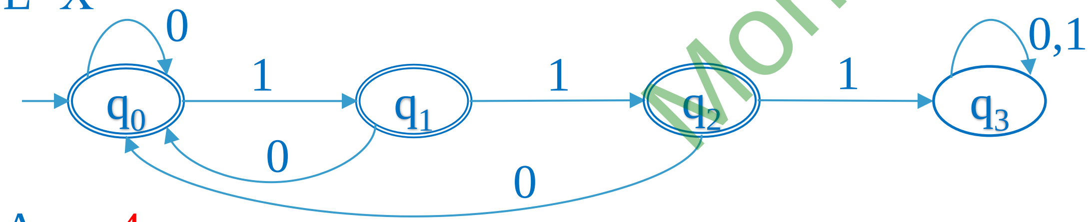
- Consider the following definition of a lexical token id for an identifier in a programming language, using extended regular expressions:
- **letter** →[A−Za−z]
- **digit** →[0−9]
- **id → letter** (**letter | digit**)*
- Which one of the following Non-deterministic Finite-state Automata with ∈-transitions accepts the set of valid identifiers? (A double-circle denotes a final state)
- Valid identifier should start with letter and followed by either letter or digit.
- (A) wrong, identifier can be start with digit.(l+ ∈)(l+d)*
- (B) wrong, because this NFA gives identifier as l(l*+d*)
- (C) correct ,l(l+d)*
- (D) wrong ,l(ld)*
- Ans : (C)



- **GATE CS 2023 | Question: 53**
- Consider the language L over the alphabet {0, 1}, given below:
- L = {w ∈ {0, 1}* | w does not contain three or more consecutive 1's}.
- The minimum number of states in a Deterministic Finite-State Automaton (DFA) for L is ____ .
- X = {w ∈ {0, 1}* | w contain three or more consecutive 1's}.



- $L=\overline{X}$

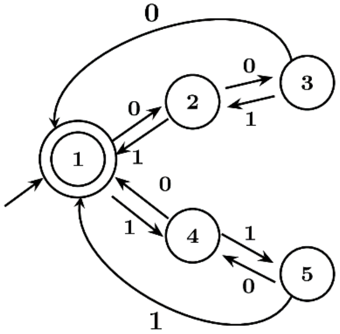


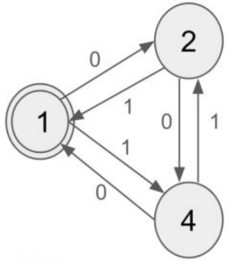
- Ans : 4

- **GATE CS 2024 | Set 1 | Question: 13**
- Let L_1,L_2 be two regular languages and L_3 a language which is not regular.
- Which of the following statements is/are always TRUE?
- (A)L_1=L_2 if and only if $L_1 \cap \overline{L_2} = \phi$
- (B)$L_1 \cup L_3$ is not regular
- (C)$\overline{L_3}$ is not regular
- (D)$\overline{L_1} \cup \overline{L_2}$ is regular
- (A) Not always true
- (B) $L_1 \cup L_3$ may be regular based on language
- (C) CFL are not closed under complement ,it may be CSL or RL but not regular ,true
- (D) Regular language are closed under complement & union ,true
- Ans: C,D

GATE CS 2024 | Set 1 | Question: 40

- Consider the 5 -state DFA. M accepting the language $L(M) \subset (0+1)^*$ shown below. For any string $w \in (0+1)^*$ let $n_0(w)$ be the number of $0's$ in w and $n_1(w)$ be the number of 1 's in w.
- Which of the following statements is/are FALSE?
- (A)States 2 and 4 are distinguishable in M
- (B)States 3 and 4 are distinguishable in M
- (C)States 2 and 5 are distinguishable in M
- (D)Any string w with $n_0(w)=n_1(w)$ is in $L(M)$



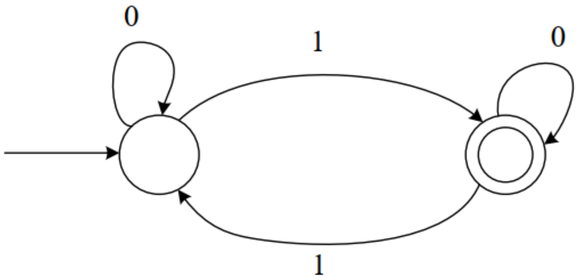


- (A) True ,State 2 and 4 are distinguishable
- (B) False ,state 3 and 4 are equal state so not distinguishable
- (C) False ,state 2 and 5 are equal state so not distinguishable
- (D) true , $n_0(w)=n_1(w)$
- Ans : B,C

GATE CS 2024 | Set 1 | Question: 51

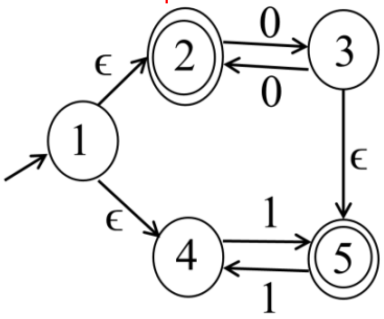
- Consider the following two regular expressions over the alphabet {0,1}:
- $r = 0^* + 1^*$
- $s = 01^* + 10^*$
- The total number of strings of length less than or equal to 5, which are neither in r nor in s, is ____________
- $|0| = \{\epsilon\}$ generate from r
- |1|={0,1} generate from r or s
- |2|={00,01,10,11} generate from r or s
- |3|={000,011,100,111} generate from r or s
- {001,010,101,110} not generate from r or s , number of strings =4=2^3-4=4
- |4|={0000,0111,1000,1111} generate from r or s
- {0001,0010, 0011, 0100,0101, 0110, 1001,1010, 1011,1100,1101,1110} not generate
- Number of strings =2^4-4=12
- Number of strings of length 5, which are neither in r nor in s= 2^5-4=28
- Total number of strings of length less than or equal to 5=4+12+28=44
- Ans : 44

- **GATE CS 2024 | Set 2 | Question: 12**
- Which one of the following regular expressions is equivalent to the language accepted by the DFA given below?



- (A) $0^*1(0 + 10^*1)^*$ (B) $0^*(10^*11)^*0^*$
- (C) $0^*1(010^*1)^*0^*$ (D) $0(1 + 0^*10^*1)^*0^*$
- L={1,01,010,00100,1011,10011…..0*1,0*10*,(0*10*1)*0*10*}
- L={odd numbers of 1 any numbers of 0}
- (A) It will generate all strings accepted by DFA
- (B) Can’t generate 1,01 ,010 …
- (C) Can’t generate 10011.
- (D) It will generate 0 which is not accepted by DFA
- Ans : (A) $0^*1(0 + 10^*1)^*$

- **GATE CS 2024 | Set 2 | Question: 31**
- Let M be the 5-state NFA with ϵ-transitions shown in the diagram below.



- Which one of the following regular expressions represents the language accepted by M ?
- (A)(00)*+1(11)* (B)0* + (1 + 0(00)*)(11)*
- (C)(00)* + (1 + (00)*)(11)* (D)0^+ + 1(11)* + 0(11)*
- L= {ϵ,0,1,00,000,011,111,0000,00011…….,0*,1(11)*,0(00)*(11)*}
- (A) Odd 0s,011,00011 can't be generated.
- (B) It can generate all strings accepted by NFA.
- (C) Odd 0s ,00011 can't be generated .
- (D) ϵ can't be generated
- Ans : (B)0* + (1 + 0(00)*)(11)*

- **GATE CS 2024 | Set 2 | Question: 52**
- Let L_1 be the language represented by the regular expression $b^*ab^*(ab^*ab^*)^*$ and $L_2 = \{w \in (a+b)^* \mid |w| \leq 4\}$, where $|w|$ denotes the length of string w. The number of strings in L_2 which are also in L_1 is ________.
- L_2={∈,a,b,aa,ab,ba,bb,……………bbbb}
- $L_1 = b^*ab^*(ab^*ab^*)^*$
- Strings in L_2 which are also in L_1 is
- |1|={a}
- |2|={ab,ba}
- |3|={aaa,bab,bba,abb}
- |4|=with 1 a={bbba,abbb,bbab,babb}
- with 3 a={aaab,aaba,abaa,baaa}
- Total =1+2+4+8=15
- Ans: 15