Algorithms
Chapter 2: Brute Force

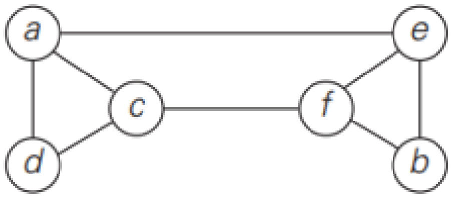
GATE CS PYQ
Solved by Monalisa

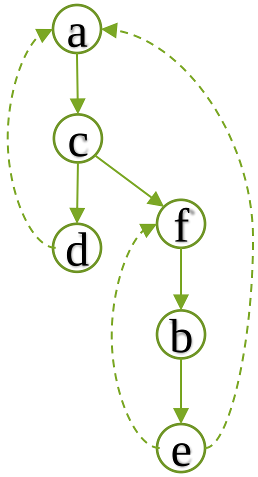
Section 5: Algorithms

- Searching, sorting, hashing. Asymptotic worst case time and space complexity. Algorithm design techniques : greedy, dynamic programming and divide-and-conquer . Graph traversals, minimum spanning trees, shortest paths
- Chapter 1:Algorithim Analysis:-Algorithm intro , Order of growth ,Asymptotic notation, Time complexity, space complexity, Analysis of Recursive & non recursive program, Master theorem]
- Chapter 2:Brute Force:-Sequential search, Selection Sort and Bubble Sort , Radix sort, Depth first Search and Breadth First Search.
- Chapter 3: Decrease and Conquer :- Insertion Sort, Topological sort,Binary Search .
- Chapter 4: Divide and conquer:-Min max problem , matrix multiplication ,Merge sort ,Quick Sort , Binary Tree Traversals and Related Properties .
- Chapter 5: Transform and conquer:- Heaps and Heap sort, Balanced Search Trees.
- Chapter 6: Greedy Method:-knapsack problem , Job Assignment problem, Optimal merge, Hoffman Coding, minimum spanning trees, Dijkstra's Algorithm.
- Chapter 7: Dynamic Programming:-The Bellman-Ford algorithm ,Warshall's and Floyd's Algorithm ,Rod cutting, Matrix-chain multiplication ,Longest common subsequence ,Optimal binary search trees
- Chapter 8: Hashing.
- Reference : Introduction to Algorithms by Thomas H. Cormen
- Introduction to the Design and Analysis of Algorithms, by Anany Levitin
- My Note

GATE CS 2000 | Question: 19

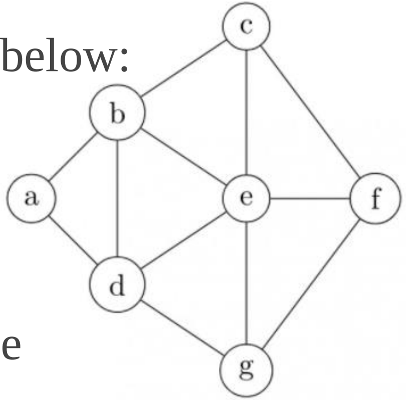
- Let G be an undirected graph. Consider a depth-first traversal of G, and let T be the resulting depth-first search tree. Let u be a vertex in G and let v be the first new (unvisited) vertex visited after visiting u in the traversal. Which of the following statements is always true?
  (A) {u,v} must be an edge in G, and u is a descendant of v in T
  (B) {u,v} must be an edge in G, and v is a descendant of u in T
  (C) If {u,v} is not an edge in G then u is a leaf in T
  (D) If {u,v} is not an edge in G then u and v must have the same parent in T.





- DFS traversal with time stamp
- $a_{1/12}$,$c_{2/11}$, $d_{3/4}$, $f_{5/10}$, $b_{6/9}$, $e_{7/8}$
- Let u=d,v=f
- A) no edge between d and f,and in tree d is not descendant of f.False
- B) no edge between d and f,and in tree f is not descendant of u.False
- C) d is a leaf so true.
- D)Always not same parent they may have same ancestor.False
- Ans : **(C)** If {u,v} is not an edge in G then u is a leaf in T

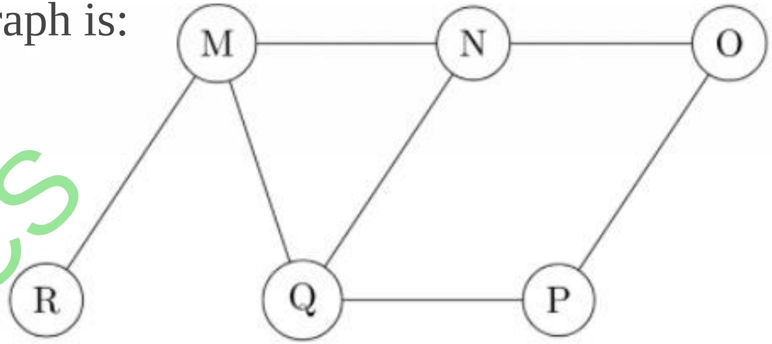
- GATE IT 2008 | Question: 47
- Consider the following sequence of nodes for the undirected graph given below:



- 1.a b e f d g c
- 2.a b e f c g d
- 3.a d g e b c f
- 4.a d b c g e f
- A Depth First Search (DFS) is started at node a. The nodes are listed in the order they are first visited. Which of the above is/are possible output(s)?
- (A) 1 and 3 only, (B) 2 and 3 only (C)2 , 3 and 4 only(D) 1,2and 3 only
- 1. a_1, b_2, e_3, f_4, d g c ,from f it can either visit c or g not d ,not possible.
- 2. $a_{1/14}$, $b_{2/13}$, $e_{3/12}$, $f_{4/11}$, $c_{5/6}$,$g_{7/10}$,$d_{8/9}$, possible.
- 3. $a_{1/14}$, $d_{2/13}$, $g_{3/12}$, $e_{4/11}$, $b_{5/10}$, $c_{6/9}$, $f_{7/8}$, possible.
- 4.a_1, d_2, b_3, c_4, g e f ,from c it can either visit e or f not g , not possible.
- Ans : (B) 2 and 3 only

- GATE CS 2008 | Question: 7
- The most efficient algorithm for finding the number of connected components in an undirected graph on *n* vertices and *m* edges has time complexity
- (A) $\Theta(n)$ (B) $\Theta(m)$ (C) $\Theta(m+n)$ (D) $\Theta(mn)$
- **Applications of both DFS &BFS** include checking connectivity and finding number of connected component .
- Both DFS & BFS have same time complexity .
- For the *adjacency matrix representation*, the traversal time is in $\boldsymbol{\Theta(n^2)}$, and for the *adjacency list representation*, it is in $\boldsymbol{\Theta(n + m)}$.
- **Ans :** (C) $\Theta(m+n)$

- GATE CS 2008 | Question: 19
- The Breadth First Search algorithm has been implemented using the queue data structure. One possible order of visiting the nodes of the following graph is:



- (A) MNOPQR (B) NQMPOR
- (C)QMNPRO (D)QMNPOR
- (A) MNOPQR ,QR should come before O , Wrong.
- (B) NQMPOR ,O should come before P, Wrong.
- (C) QMNPRO is correct BFS sequence .
- (D) QMNPOR ,R should come before O , Wrong.
- Ans : (C)QMNPRO

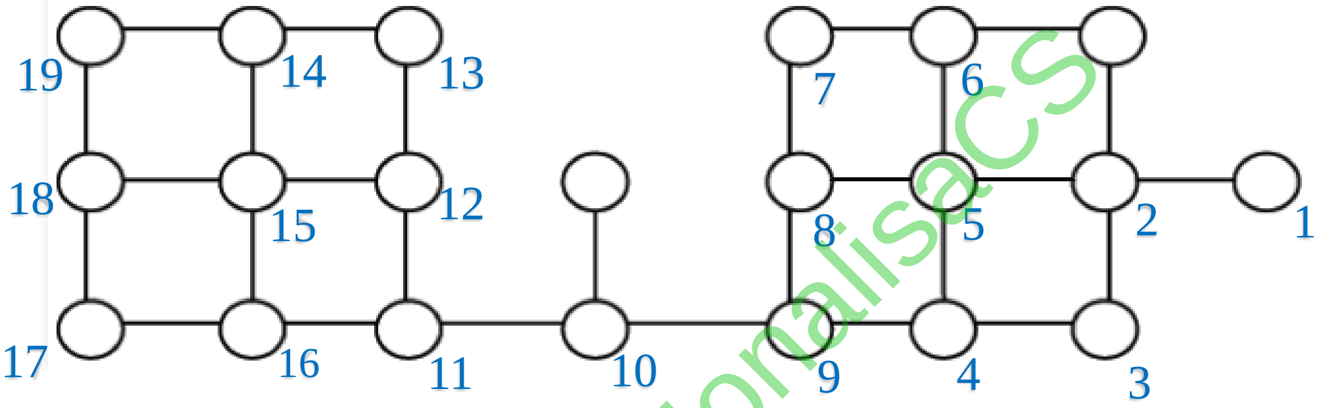
- GATE CS 2009 | Question: 11
- What is the number of swaps required to sort n elements using selection sort, in the worst case?
- (A) $\Theta(n)$
- (B) $\Theta(nlogn)$
- (C) $\Theta(n^2)$
- (D) $\Theta(n^2logn)$
- We have 1 swap in each loop.
- The number of key swaps is only $\Theta(n)$, or, more precisely, $n - 1$.
- Ans : (A) $\Theta(n)$

- GATE CS 2013 | Question: 6
- Which one of the following is the tightest upper bound that represents the number of swaps required to sort n numbers using selection sort?
- (A) O(log n)
- (B) O(n)
- (C) O($n$log$n$)
- (D) O(n^2)
- We scan the entire given list to find its smallest element and exchange it with the position, putting the smallest element in its final position in the sorted list.
- In selection max we can do is $n - 1$ swaps .
- n-1=O(n) [Tightest upper bound]
- Ans: (B) O(n)

- GATE CS 2014,Set-1| Question :11
- Let G be a graph with n vertices and m edges. What is the tightest upper bound on the running time of Depth First Search on G, when G is represented as an adjacency matrix?
- **(A)** O(n) **(B)** O(m+n) **(C)** $O(n^2)$ **(D)** O(mn)
- For the *adjacency matrix representation*, the traversal time is in $\Theta(|V|^2)$,and for the *adjacency list representation*, it is in $\Theta(|V| + |E|)$ where $|V|$ and $|E|$ are the number of the graph's vertices and edges, respectively .
- $|V|$**=n** , $|E|$**=m**
- Tightest upper bound on the running time of Depth First Search on G is represented as an adjacency matrix $O(n^2)$
- Ans : **(C)** $O(n^2)$

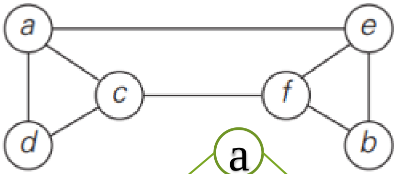
- GATE CS 2014,Set-2|Question :14
- Consider the tree arcs of a BFS traversal from a source node W in an unweighted, connected, undirected graph. The tree T formed by the tree arcs is a data structure for computing.
- **(A)** the shortest path between every pair of vertices.
- **(B)** the shortest path from W to every vertex in the graph.
- **(C)** the shortest paths from W to only those nodes that are leaves of T.
- **(D)** the longest path in the graph
- One of the application of BFS algorithm is to find the shortest path
- In the given question the BFS algorithm starts from the source vertex W and we can find the shortest path from W to every vertex of the graph.
- Ans: **(B)** the shortest path from W to every vertex in the graph.

- GATE CS 2014,Set-3|Question 13:
- Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is __________.

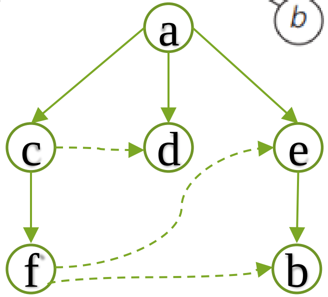


- Ans :19

- GATE CS 2015 Set 1 | Question: 45
- Let G=(V,E) be a simple undirected graph, and *s* be a particular vertex in it called the source. For $x \in V$, let *d(x)* denote the shortest distance in G from *s* to *x*. A breadth first search (BFS) is performed starting at *s*. Let T be the resultant BFS tree. If (*u*,*v*) is an edge of *G* that is not in *T*, then which one of the following CANNOT be the value of d(*u*)−d(*v*)?



- (A) −1 (B) 0 (C)1 (D)2
- cd,ef,bf are edges of G that are not in T.

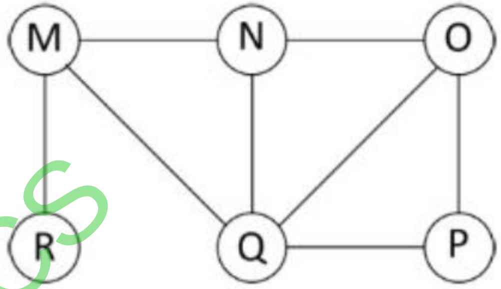


- d(c)-d(d)=1-1=0
- d(f)-d(e)=2-1=1
- d(e)-d(f)=1-2=-1
- d(b)-d(f)=2-2=0
- (A) -1, (B) 0, (C) 1 are possible
- (D) d(u)−d(v)=2 is not possible. This is because on BFS traversal we either visit u first or v. Let's take u first. Now, we put all neighbors of u on queue. Since v is a neighbor and v is not visited before as assumed, d(v) will become d(u)+1. Similarly, for v being visited first.
- Ans : (D) 2

- GATE CS 2016,Set-2|Question 11:Breadth First Search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex t at a distance four from the root. If t is the n-th vertex in this BFS traversal, then the maximum possible value of n is ____.
- Max number of nodes = $2^{(h+1)} - 1 = 2^{(4+1)} - 1 = 32\text{-}1\text{=}31$
- At distance four, last node is 31.
- Ans :31

- GATE CS 2016,Set-2 |Question 41:In an adjacency list representation of an undirected simple graph G = (V,E), each edge (u,v) has two adjacency list entries: [v] in the adjacency list of u, and [u] in the adjacency list of v. These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If |E| = m and |V| = n, and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list?
- **(A)** $\Theta(n^2)$ **(B)** $\Theta(m+n)$ **(C)** $\Theta(m^2)$ **(D)** $\Theta(n^4)$
- First find twins of each node.
- You can do this using **level order traversal (i.e., BFS)** once.
- Time complexity of BFS is $\Theta(m+n)$ for adjacency list representation.
- Final, time complexity is $\Theta(m + n)$ to set twin pointer.
- Ans : **(B)** $\Theta(m+n)$

- GATE CS 2017,Set-2 |Question 15:The Breath First Search(BFS) algorithm has been implemented using queue data structure. Which one of the following is a possible order of visiting the nodes in the graph below.



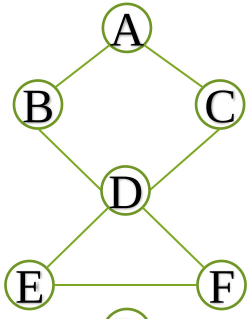
- **(A)** MNOPQR **(B)** NQMPOR
  (C) QMNROP **(D)** POQNMR

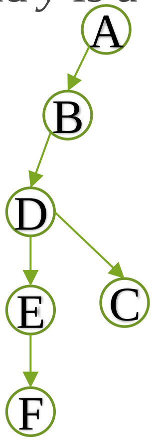
- (A) MNOPQR – O is not adjacent of M,so MNO false.
- (B) NQMPOR – P is not adjacent of N ,so NQMP is false.
- (C) QMNROP – R is not the adjacent of Q, so QMNR is false.
- (D) POQNMR – is the correct sequence. Hence Option (D).
- Ans: **(D)** POQNMR

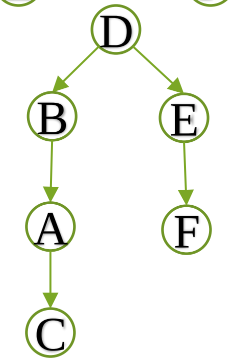
- GATE CS 2018 |Question 30:Let G be a simple undirected graph. Let T_D be a depth first search tree of G. Let T_B be a breadth first search tree of G. Consider the following statements.
- (I) No edge of G is a cross edge with respect to T_D.
  (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D.)
- (II) For every edge (u,v) of G, if u is at depth i and v is at depth j in T_B, then $|i-j| = 1$.
- Which of the statements above must necessarily be true?
- (A) I only (B) II only (C) Both I and II (D) Neither I nor II
- I. Undirected graph do not have cross edges in DFS. But can have cross edges in directed graph. Hence True.
- II. Just draw a triangle ABC. Source is A. Vertex B and C are at same level at distance 1.
- There is an edge between B and C too. So here $|i - j| = |1 - 1| = 0$. Hence, False.
- Ans: (A) I only

GATE CS 2021 Set 1 | Question: 41

- An *articulation point* in a connected graph is a vertex such that removing the vertex and its incident edges disconnects the graph into two or more connected components.
- Let *T* be a *DFS* tree obtained by doing *DFS* in a connected undirected graph *G*.
- Which of the following options is/are correct ?
- (A) Root of *T* can never be an articulation point in *G*.
- (B) Root of *T* is an articulation point in *G* if and only if it has 2 or more children.
- (C) A leaf of *T* can be an articulation point in *G*.
- (D) If *u* is an articulation point in *G* such that *x* is an ancestor of *u* in *T* and *y* is a descendent of *u* in *T*, then all paths from *x* to *y* in *G* must pass through *u*.

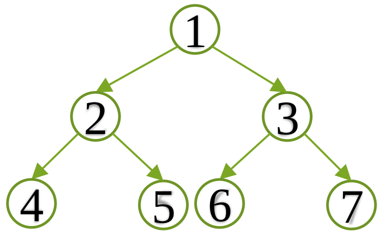






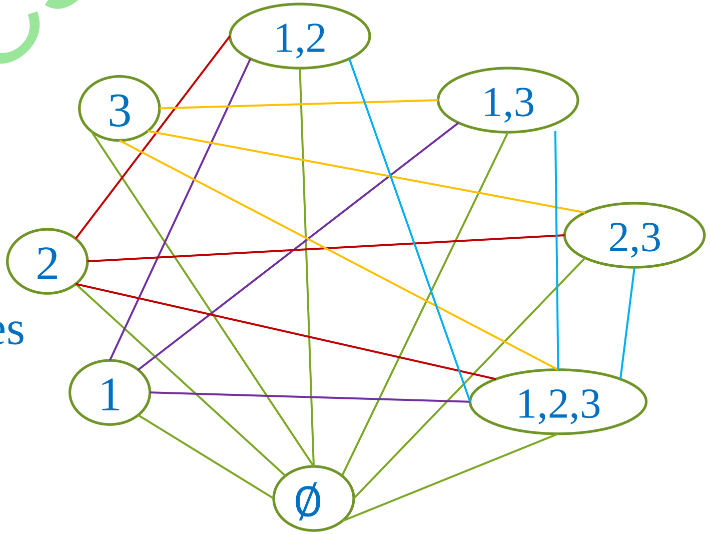
- (A)Root of T can be articulation point ,false.
- (B)True
- (C)Leaf can’t be articulation point ,false
- (D)let u=D articulation point.
- A,B are ancestor of D,while C,E,F are descendent
- Let x=B,y=C,u=D
- Then all paths from x to y in G are not passing through u.
- False
- Ans: (B)

- GATE CS 2021 Set 2 | Question: 16:
- Consider a complete binary tree with 7 nodes. Let A denote the set of first 3 elements obtained by performing Breadth-First Search (BFS) starting from the root. Let B denote the set of first 3 elements obtained by performing Depth-First Search (DFS) starting from the root.
- The value of |A−B| is ________



- BFS: 1,2,3,4,5,6,7
- DFS: 1,2,4,5,3,6,7
- A={1,2,3} B={1,2,4}
- A-B={3}
- |A-B|=1
- Ans :1

- **GATE CS 2023 | Question: 46**
- Let U = {1, 2, 3}. Let 2^U denote the powerset of U. Consider an undirected graph G whose vertex set is 2^U. For any A,B ∈ 2^U, (A,B) is an edge in G if and only if (i) A≠ B , and (ii) either A⊆B or B⊆A . For any vertex A in G, the set of all possible orderings in which the vertices of G can be visited in a Breadth First Search (BFS) starting from A is denoted by B(A). If Ø denotes the empty set, then the cardinality of B(Ø) is ________ .
- Given, U = {1,2,3}
- V = {Ø,1,2,3, (1,2) , (1,3) , (2,3) , (1,2,3)}
- |V|=8
- Ø is proper subset to every other set in the vertex set.
- Ø is connected with 7 vertices directly, these 7 vertices can be visited in any order in BFS.
- |B(Ø)| = 7! = 5040.
- Ans : 5040



- **GATE DA 2024 | Question: 4**
- Consider performing depth-first search (DFS) on an undirected and unweighted graph G starting at vertex s. For any vertex u in G, $d[u]$ is the length of the shortest path from s to u. Let (u, v) be an edge in G such that $d[u] < d[v]$. If the edge (u, v) is explored first in the direction from u to v during the above DFS, then (u, v) becomes a ______ edge.
- (A)Tree (B)Cross (C)Back (D)Gray
- (A) ***Tree edges*** are edges in the depth-first forest. Edge *(u,v)* is a tree edge if *v* was first discovered by exploring edge (u,v).
- (B) ***Back edges*** are those edges *(u,v)* connecting a vertex u to an ancestor in a depth-first tree. We consider self-loops, which may occur in directed graphs, to be back edges.
- (C) ***Cross edges*** are all other edges. They can go between vertices in the same depth-first tree, as long as one vertex is not an ancestor of the other, or they can go between vertices in different depth-first trees.
- (D) ***Gray edges*** v has already been discovered , but all the vertices that are reachable from v are not yet discovered.
- Ans: (A) Tree

- **GATE DA 2024 | Question: 34**
- Consider a state space where the start state is number 1. The successor function for the state numbered *n* returns two states numbered *n+1* and *n+2*. Assume that the states in the unexpanded state list are expanded in the ascending order of numbers and the previously expanded states are not added to the unexpanded state list.Which **ONE** of the following statements about breadth-first search (BFS) and depth-first search (DFS) is true, when reaching the goal state number 6?
- (A) BFS expands more states than DFS.
- (B) DFS expands more states than BFS.
- (C) Both BFS and DFS expand equal number of states.
- (D) Both BFS and DFS do not reach the goal state number 6.
- Ans : (C) Both BFS and DFS expand equal number of states.

- ## GATE CS 2024 | Set 1 | Question: 35

- Let G be a directed graph and T a depth first search (DFS) spanning tree in G that is rooted at a vertex v. Suppose T is also a breadth first search (BFS) tree in G, rooted at v. Which of the following statements is/are TRUE for *every* such graph G and tree T?
- (A) There are no back-edges in G with respect to the tree T
- (B) There are no cross-edges in G with respect to the tree T
- (C) There are no forward-edge in G with respect to the tree T
- (D) The only edges in G are the edges in T
- ***Back edges*** are those edges *(u,v)* connecting a vertex u to an ancestor in a depth-first tree. We consider self-loops, which may occur in directed graphs, to be back edges.
- ***Forward edges*** are those nontree edges *(u,v)* connecting a vertex u to a descendant in a depth-first tree.
- ***Cross edges*** are all other edges. They can go between vertices in the same depth-first tree, as long as one vertex is not an ancestor of the other, or they can go between vertices in different depth-first trees.
- DFS=BFS so back edge ,cross edge possible but no forward edge
- If forward edge present, then DFS≠ BFS
- (D) there may be more edges in graph
- Ans : (C) There are no forward-edge in G with respect to the tree T

- **GATE CS 2024 | Set 2 | Question: 25**
- Let A be an array containing integer values. The distance of A is defined as the minimum number of elements in A that must be replaced with another integer so that the resulting array is sorted in non-decreasing order. The distance of the array [2,5,3,1,4,2,6] is ___________.
- A=[2,5,3,1,4,2,6]
- A=[2,2,3,1,4,2,6]
- A=[2,2,3,3,4,2,6]
- A=[2,2,3,3,4,4,6]
- 3 replace So distance=3
- Ans :3