Theory of Computation
Chapter 2: Context Free Language

GATE CS Previous year Questions
Chapter wise Solved By
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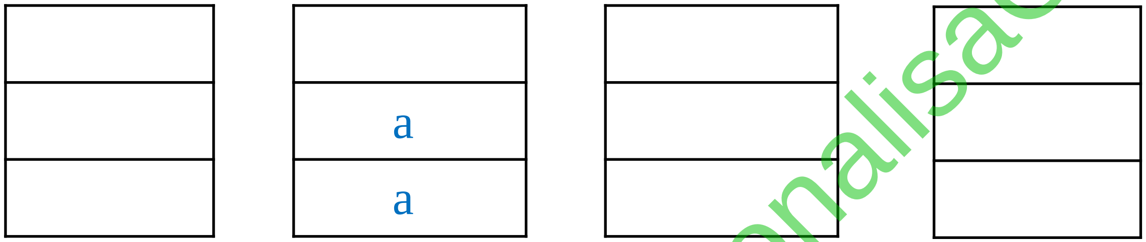
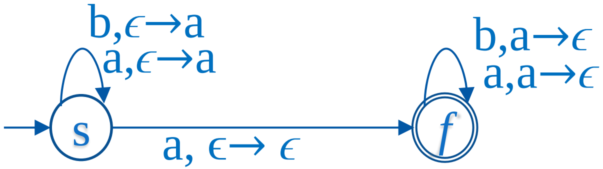
- GATE IT 2004,Q 40 : Let M=(K , Σ , Γ , Δ , s , F) be a pushdown automaton, where K=(s , f),F={f},Σ={a,b},Γ={a} and
- $\Delta=\{((s,a,\epsilon), (s,a)), ((s,b,\epsilon), (s,a)), ((s,a,\epsilon), (f,\epsilon)) , ((f,a,a), (f,\epsilon)), ((f,b,a), (f,\epsilon))\}$
- Which one of the following strings is not a member of L(M)?
- **(A)** aaa **(B)** aabab **(C)** baaba **(D)** bab
- L={Any odd length string having 'a' at middle}





(A) aaa — Accept

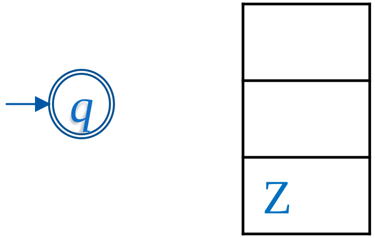
(B) aabab — Not Accept by Empty stack

(C) baaba — Accept

(D) bab — Accept

- By Final state mechanism all string accept
- While by empty stack mechanism option B not accept
- Ans: **(B)** aabab

- GATE IT 2005,Q 38: Let P be a non-deterministic push-down automaton (NPDA) with exactly one state, q, and exactly one symbol, Z, in its stack alphabet. State q is both the starting as well as the accepting state of the PDA. The stack is initialized with one Z before the start of the operation of the PDA. Let the input alphabet of the PDA be Σ. Let L(P) be the language accepted by the PDA by reading a string and reaching its accepting state. Let N(P) be the language accepted by the PDA by reading a string and emptying its stack.
- Which of the following statements is TRUE?
- **(A)** L(P) is necessarily Σ* but N(P) is not necessarily Σ*
  (B) N(P) is necessarily Σ* but L(P) is not necessarily Σ*
  (C) Both L(P) and N(P) are necessarily Σ*
  (D) Neither L(P) nor N(P) are necessarily Σ*



- It is ALREADY in the FINAL state we may not have ANY transition for any input symbol. In this case string will be rejected as it will never finish reading the string
- It may be the case that the string has a dead configuration over the transitions of the PDA i.e. PDA does not have a transition for a particular alphabet or string.
- Hence it does not accept all the strings over Σ*.
- **Ans :(D)** Neither L(P) nor N(P) are necessarily Σ*

- GATE CS 2010,Q40: Consider the languages

  $L1 = \{0^i1^j \mid i \mathrel{!=} j\}$.

  $L2 = \{0^i1^j \mid i = j\}$.

  $L3 = \{0^i1^j \mid i = 2j+1\}$.

  $L4 = \{0^i1^j \mid i \mathrel{!=} 2j\}$.

  (A) Only L2 is context free

  (B) Only L2 and L3 are context free

  (C) Only L1 and L2 are context free

  (D) All are context free
- All languages L1, L2, L3 and L4 has only one comparison and it can be accepted by PDA (single stack), hence all are Context Free Languages
- **Ans :**(D) All are context free

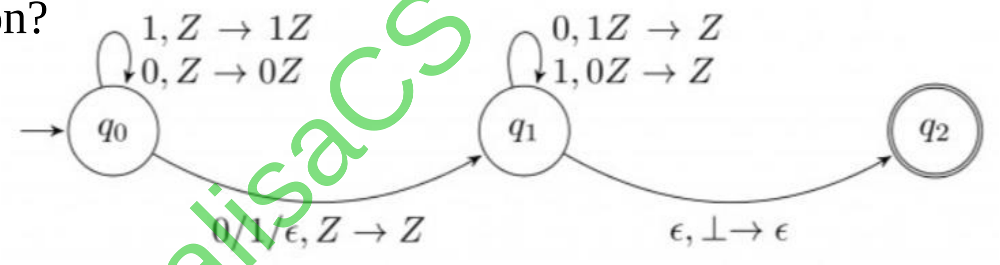
- GATE CS 2011,Q24: Let P be a regular language and Q be a context-free language such that Q⊆P. (For example, let P be the language represented by the regular expression $p*q*$ and Q be $\{p^n q^n | n \in N\}$). Then which of the following is **ALWAYS** regular?
- (A) $P \cap Q$ (B) $P - Q$ (C) $\Sigma^* - P$ (D) $\Sigma^* - Q$
- A. $P \cap Q = Q$, as $Q \subseteq P$, hence context free but not regular.
- B. $P - Q = P \cap \overline{Q}$ ≠CFL. CFL is not closed under complement.
- C. $\Sigma* - P = \overline{P}$, hence regular. RL is closed under complement
- D. $\Sigma* - Q = \overline{Q}$ ≠CFL. CFL is not closed under complement.
- **Ans :** (C) $\Sigma^* - P$

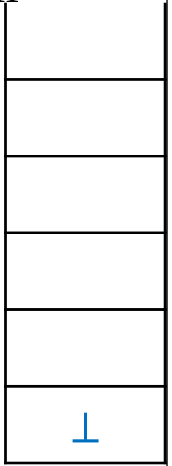
- GATE CS 2011,Q26: Consider the languages L1,L2andL3 as given below.
- $L1=\{0^p1^q|p,q\in N\}$,
- $L2=\{0^p1^q|p,q\in N \text{ and } p=q\}$and
- $L3=\{0^p1^q0^r|p,q,r\in N \text{and } p=q=r\}$.
- Which of the following statements is NOT TRUE?
- (A) Push Down Automata (PDA) can be used to recognize L1 and L2
- (B) L1 is a regular language
- (C) All the three languages are context free
- (D) Turing machines can be used to recognize all the languages
- L1: regular language
- L2: context free language
- L3: context sensitive language
- A.RL⊂CFL,So PDA can be used to recognize RL & CFL.True
- B.True
- C.False
- D.RL ⊂CFL ⊂CSL ⊂REL
- So TM can be used recognize all Language.True
- Ans: (C) All the three languages are context free

- GATE CS 2013,Q32: Consider the following languages.
- $L_1=\{0^p1^q0^r \mid p,q,r \geq 0\}$
- $L_2=\{0^p1^q0^r \mid p,q,r \geq 0, p \neq r\}$
- Which one of the following statements is **FALSE**?
- (A)L_2 is context-free.
- (B)$L_1 \cap L_2$ is context-free.
- (C)Complement of $L_2$is recursive.
- (D)Complement of L_1 is context-free but not regular.
- L_1 = 0*1*0* Regular Language .
- L_2 we have one comparison (i.e. $p \neq r$) so it is CFL.
- (A)True
- (B)$L_1 \cap L_2$ is context-free. (by closure properties) True
- (C)L_2 is CFL and CFL is not closed under complement , complement L_2 = CSL .Every CSL is recursive so Complement of L_2 is recursive. True
- (D)Regular language is closed under complement so complement of L_1 must be regular .False
- Ans:(D)Complement of L_1 is context-free but not regular.

- GATE CS 2014,Set-3,Q36: Consider the following languages over the alphabet $\sum=\{0,1,c\}$
- $L_1=\{0^n1^n|n\geq 0\}$
- $L_2=\{wcw^r|w\in\{0,1\}^*\}$
- $L_3=\{ww^r|w\in\{0,1\}^*\}$
- Here, w^r is the reverse of the string w. Which of these languages are deterministic Context-free languages?
- (A)None of the languages (B)Only L_1
- (C)Only L_1 and L_2 (D)All the three languages
- L_1 and L_2 are DCFL, as we can design DPDA for them.
- L_3, we cannot make DPDA for it, as we cannot locate the middle of string, so DPDA for L_3 is not possible. It can be accepted by NPDA only, so L_3 is CFL but not DCFL.
- Ans: (C)Only L_1 and L_2

- GATE CS 2015,Set-1,Q51: Consider the NPDA ⟨Q = {q0, q1, q2}, Σ = {0, 1}, Γ = {0, 1, ⊥}, δ, q0, ⊥, F = {q2}⟩, where (as per usual convention) Q is the set of states, Σ is the input alphabet, Γ is stack alphabet, δ is the state transition function, q0 is the initial state, ⊥ is the initial stack symbol, and F is the set of accepting states, The state transition is as follows:
- Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?
- **(A)** 10110 **(B)** 10010
- **(C)** 01010 **(D)** 01001

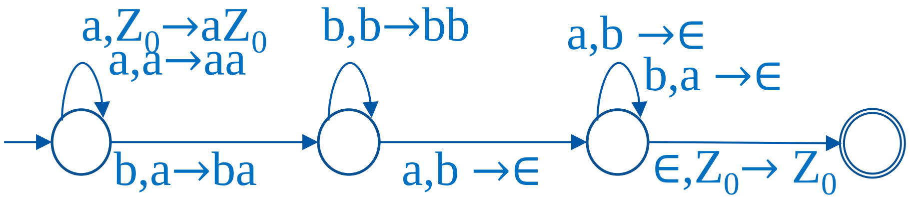




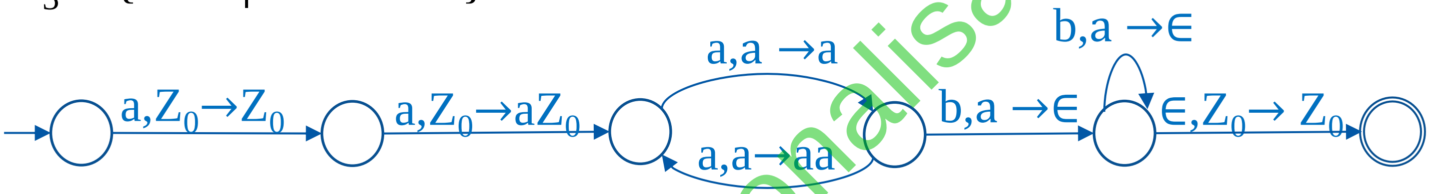
- In q_0 state for ‘1’, a ‘1’ is pushed and for a ‘0’, a ‘0’ is pushed.
- In q_1 state, for a ‘0’ a ‘1’ is popped, and for ‘1’ a ‘0’ is popped.
- L= $w0(w^r)' \cup w1(w^r)' \cup w(w^r)'$, $(w^r)'$ is the complement of w^r.
- |101100|= 6 letters and we are given 5 letter strings in option.
- So L= $w0(w^r)'$, with w = 10110.
- $(w^r)'$ = (01101)'= 10010.
- Ans: **(B)** 10010

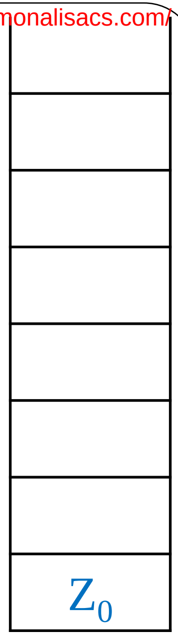
- GATE CS 2015,Set-3,Q32:Which of the following languages are context-free?
- $L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$
- $L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$
- $L_3 = \{a^m b^n \mid m = 2n + 1\}$
- **(A)** L_1 and L_2 only **(B)** L_1 and L_3 only **(C)** L_2 and L_3 only **(D)** L_3 only
- L_1: can be accepted by PDA, hence CFL.
- L_2: First push all the a's in the stack then push all the b's in the stack.
- Now again a's come which cannot be compared by previous a's in the stack because at top of the stack's there are b’s .So not CFL.
- L_3: can be accepted by PDA, hence CFL
- **Ans:(B)** L_1 and L_3 only

- GATE CS 2015,Set-3,Q32:Which of the following languages are context-free?
- $L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}, L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}, L_3 = \{a^m b^n \mid m = 2n + 1\}$
- $L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$



- $L_3 = \{a^m b^n \mid m = 2n+1\} = a^{2n+1} b^n$

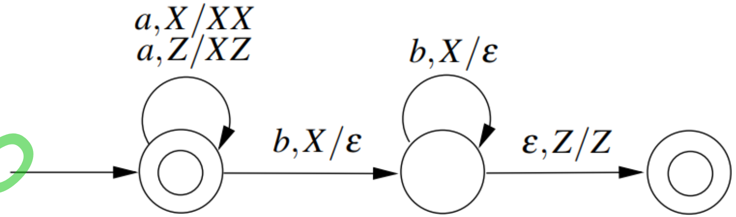




Stack

- GATE CS 2016,Set-1,Q42: Consider the following context-free grammars:
- G1:S→aS|B,B→b|bB
- G2:S→aA|bB,A→aA|B|ε,B→bB|ε
- Which one of the following pairs of languages is generated by G1 and G2 , respectively?
- (A)$\{a^m b^n | m>0 \text{ or } n>0\}$and $\{a^m b^n | m>0 \text{ and } n>0\}$
- (B)$\{a^m b^n | m>0 \text{ and } n>0\}$and $\{a^m b^n | m>0 \text{ or } n\geq 0\}$
- (C)$\{a^m b^n | m\geq 0 \text{ or } n>0\}$and $\{a^m b^n | m>0 \text{ and } n>0\}$
- (D)$\{a^m b^n | m\geq 0 \text{ and } n>0\}$and $\{a^m b^n | m>0 \text{ or } n>0\}$
- B→b|bB =b^+
- S→aS|B=$a^*b^+ + b^+ = a^*b^+$
- L(G1)=$\{a^m b^n | m\geq 0 \text{ and } n>0\}$={b,bbbb,ab,aabbb,…}
- B→bB|ε=b^*
- A→aA|B|ε=$a^*+a^*b^*+b^*=a^*b^*$
- S→aA|bB=$a(a^*b^*)+bb^*=a^+b^*+b^+ = a^+ + a^+b^+ + b^+$
- L(G2)=$\{a^m b^n | m>0 \text{ or } n>0\}$=$\{a^+, b^+$,aaabb,aaaabbbb,…}
- Ans:(D)$\{a^m b^n | m\geq 0 \text{ and } n>0\}$and $\{a^m b^n | m>0 \text{ or } n>0\}$

- GATE CS 2016,Set-1,Q43 :Consider the transition diagram of a PDA given below with input alphabet $\Sigma = \{a,b\}$ and stack alphabet $\Gamma = \{X,Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



- Which one of the following is **TRUE**?
- (A)$L=\{a^nb^n|n\geq0\}$ and is not accepted by any finite automata
- (B)$L=\{a^n|n\geq0\}\cup\{a^nb^n|n\geq0\}$and is not accepted by any deterministic PDA
- (C)L is not accepted by any Turing machine that halts on every input
- (D)$L=\{a^n|n\geq0\}\cup\{a^nb^n|n\geq0\}$and is deterministic context-free
- Since initial is final state, so it can accept any number of a's or ∈ by using Acceptance by Final State .$L=\{a^n|n\geq0\}$
- by second final state it accept equal number of a's followed by equal number of b's.$L= \{a^nb^n|n\geq0\}$
- $L = \{a^n \mid n\geq0\} \cup \{ a^n b^n \mid n\geq0\}$.
- Ans:(D)$L=\{a^n|n\geq0\}\cup\{a^nb^n|n\geq0\}$and is deterministic context-free

- GATE CS 2016,Set-2,Q43 :Consider the following languages:
  $L_1 = \{a^n b^m c^{n+m} : m,n \geq 1\}$
  $L_2 = \{a^n b^n c^{2n} : n \geq 1\}$
- Which one of the following is **TRUE**?
  (A) Both $L1$ and $L2$ are context-free.
  (B) $L1$ is context-free while $L2$ is not context-free.
  (C) $L2$ is context-free while $L1$ is not context-free.
  (D) Neither $L1$ nor $L2$ is context-free.
- L_1 can be recognized by PDA, we have to push a's and b's in stack and when c's comes then pop every symbol from stack for each c's. Hence, it is CFL.
- But L_2 can't be recognized by PDA.It has 2 comparison.
- 1st comparison:number of a's = number of b's
- 2nd comparison:number of c's must be two times number of a's or b's
- It is CSL.
- Ans:(B) $L1$ is context-free while $L2$ is not context-free.

- GATE CS 2016,Set-2,Q45: Which one of the following grammars is free from left recursion?

(A) $S \rightarrow AB$
$A \rightarrow Aa \mid b$
$B \rightarrow c$

(B) $S \rightarrow Ab \mid Bb \mid c$
$A \rightarrow Bd \mid \epsilon$
$B \rightarrow e$

(C) $S \rightarrow Aa \mid B$
$A \rightarrow Bb \mid Sc \mid \epsilon$
$B \rightarrow d$

(D) $S \rightarrow Aa \mid Bb \mid c$
$A \rightarrow Bd \mid \epsilon$
$B \rightarrow Ae \mid \epsilon$

- A: has direct left recursion due to A->Aa.
- B: doesn't have any left recursion (neither direct nor indirect)
- C: has indirect left recursion due to S-> Aa and A->Sc.
- D: has indirect left recursion due to A-> Bd and B-> Ae
- Ans: (B) $S \rightarrow Ab \mid Bb \mid c$
- $A \rightarrow Bd \mid \epsilon$
- $B \rightarrow e$

- GATE CS 2016,Set-2,Q46: A student wrote two context-free grammars G1 and G2 for generating a single C-like array declaration. The dimension of the array is at least one. For example, int a[10][3];
- The grammars use D as the start symbol, and use six terminal symbols int ; id [ ] num.
- Grammar G1

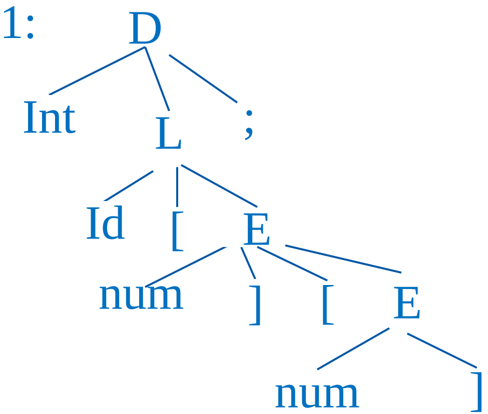
$D \rightarrow$ int L;
$L \rightarrow$ id [E
$E \rightarrow$ num]
$E \rightarrow$ num] [E

Grammar G2

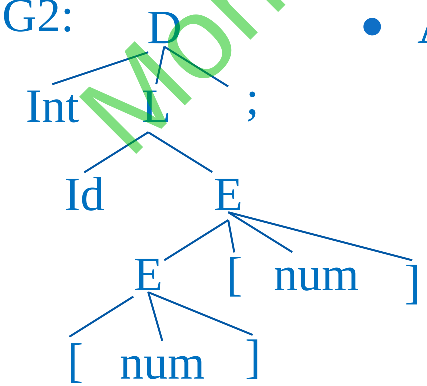
$D \rightarrow$ int L;
$L \rightarrow$ id E
$E \rightarrow$ E[num]
$E \rightarrow$ [num]

- Which of the grammars correctly generate the declaration mentioned above?
- **(A)** Both G1 and G2 **(B)** Only G1 **(C)** Only G2 **(D)** Neither G1 nor G2

G1:



G2:



- Ans: **(A)** Both G1 and G2

- GATE CS 2017,Set-1,Q10: Consider the following context-free grammar over the alphabet $\sum = \{a, b, c\}$ with S as the start symbol:
- $S \rightarrow abScT \mid abcT$
- $T \rightarrow bT \mid b$
- Which of the following represents the language generated by the above grammar?

  (A) $\{(ab)^n(cb)^n \mid n \geq 1\}$

  (B) $\{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n, m_1, m_2, \ldots\ldots, m_n \geq 1\}$

  (C) $\{(ab)^n(cb^m)^n \mid m,n \geq 1\}$

  (D) $\{(ab)^n(cb^n)^m \mid m, n \geq 1\}$
- $T \rightarrow bT \mid b = b^+$
- $S \rightarrow abcT = abcb^+$
- $S \rightarrow abScT \rightarrow ab\ abScT\ cT \rightarrow ab\ ab\ abcT\ cT\ cT = (ab)^3\ cb^+\ cb^+\ cb^+$
- $= (ab)^n\ cb^+\ cb^+\ \ldots\ldots cb^+$[n time c]
- $L = \{(ab)^n\ cb^{(m_1)}\ cb^{(m_2)} \ldots cb^{(m_n)} \mid n, m_1, m_2, \ldots, m_n \geq 1\}$
- Ans: **(B)** $\{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n, m_1, m_2, \ldots\ldots, m_n >= 1$

- GATE CS 2017,Set-1,Q34: If G is a grammar with productions
- $S \rightarrow SaS \mid aSb \mid bSa \mid SS \mid \epsilon$
- where S is the start variable, then which one of the following strings is not generated by G?
- **(A)** abab **(B)** aaab **(C)** abbaa **(D)** babba

(A)
- $S \rightarrow aSb$
- $\rightarrow abSab$
- $\rightarrow ab\ \epsilon\ ab$
- $\rightarrow ab\ ab$

(B)
- $S \rightarrow SS$
- $\rightarrow SaSS$
- $\rightarrow \epsilon aSS$
- $\rightarrow aSaSS$
- $\rightarrow a\epsilon aSS$
- $\rightarrow aa\epsilon S$
- $\rightarrow aa\ aSb$
- $\rightarrow aa\ a\epsilon b$
- $\rightarrow aa\ ab$

(C)
- $S \rightarrow SS$
- $\rightarrow aSbS$
- $\rightarrow a\epsilon bS$
- $\rightarrow ab\ bSa$
- $\rightarrow abb\ SaS\ a$
- $\rightarrow abb\epsilon aSa$
- $\rightarrow abba\epsilon a$
- $\rightarrow abbaa$

(D)
- $S \rightarrow bSa$
- $\rightarrow baSba$

- $S \rightarrow SS$
- $\rightarrow bSaS$
- $\rightarrow b\epsilon aS$
- $\rightarrow babSa$

- $L=\{n_a(w)\geq n_b(w) \mid w\in(a+b)^*\}$
- **Ans: (D)** babba

- GATE CS 2017,Set-1,Q37: Consider the context-free grammars over the alphabet {a, b, c} given below. S and T are non-terminals.
- G_1: $S \rightarrow aSb|T$, $T \rightarrow cT|\epsilon$
  G_2: $S \rightarrow bSa|T$, $T \rightarrow cT|\epsilon$
- The language $L(G_1) \cap L(G_2)$ is
- **(A)** Finite **(B)** Non-finite but regular
  (C) Context-free but not regular **(D)** Recursive but not context-free
- Strings generated by G_1:{ϵ, c, cc, ccc, … ab, aabb, aaabbb….acb, accb… aacbb, …}
- $L(G_1) = \{a^n c^m b^n | m,n \geq 0\}$
- Strings generated by G_2:{ϵ, c, cc, ccc, … ba, bbaa, bbbaaa….bca, bcca… bbcaa,…}
- $L(G_2) = \{b^n c^m a^n | m,n \geq 0\}$
- $L(G_1) \cap L(G_2) = \{\epsilon, c, cc, ccc…\} = \{c^m | m \geq 0\} = c^*$
- $L(G_1) \cap L(G_2)$ is "Not finite but regular".
- Ans: **(B)** Non-finite but regular

- GATE CS 2017,Set-1,Q38 :Consider the following languages over the alphabet $\Sigma = \{a, b, c\}$.Let $L_1 = \{a^n b^n c^m \mid m,n \geq 0\}$ and $L_2 = \{a^m b^n c^n \mid m,n \geq 0\}$
- Which of the following are context-free languages?
- I. $L_1 \cup L_2$ II. $L_1 \cap L_2$
- **(A)** I only **(B)** II only **(C)** I and II **(D)** Neither I nor II
- $L_1 = \{\epsilon, c, cc, \ldots, ab, aabb, \ldots, abc, abcc, \ldots, aabbc, aabbcc, \ldots\}$
- $L_2 = \{\epsilon, a, aa, \ldots, bc, bbcc, \ldots, abc, aabc, \ldots, abbcc, aabbcc, \ldots\}$
- $L_1 \cup L_2 = \{\epsilon, a, aa, .., c, cc, .. ab, bc, \ldots, aabb, bbcc, .., abc, abcc, aabc, \ldots\}$
- $L_1 \cup L_2 = \{a^n b^n c^m \cup a^m b^n c^n \mid m,n \geq 0\}$ CFL.
- $L_1 \cap L_2 = \{\epsilon, abc, aabbcc, aaabbbccc, \ldots\}$
- $L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$ CSL.
- CFL is not closed under Intersection and its closed under Union.
- **Ans: (A)** I only

- GATE CS 2017,Set-2,Q4:Let L_1,L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?
- I.$L_1 \cup L_2$ is context-free.
- II.$\overline{L_1}$ is context-free.
- III.$L_1 - R$ is context-free
- IV.$L_1 \cap L_2$ is context-free
- (A)I, II and IV only (B)I and III only (C)II and IV only (D)I only
- I.CFL is closed under Union ,So $L_1 \cup L_2$ is CFL, CORRECT.
- II.CFL is not closed under Complement, So $\overline{L_1}$ is CFL, INCORRECT.
- III.$L_1 - R = L_1 \cap \bar{R}$,Regular language is closed under compliment.
- $\bar{R}$ =RL ,Regular language is closed under intersection with any language,
- $L \cap R$ =L.So $L_1 - R$ is context free. CORRECT
- IV.CFL is not closed under Intersection.
- So $L_1 \cap L_2$ is CFL , INCORRECT
- Ans: (B)I and III only

- GATE CS 2017,Set-2,Q16: Identify the language generated by the following grammar, where S is the start variable.
- $S \rightarrow XY$
- $X \rightarrow aX|a$
- $Y \rightarrow aYb|\epsilon$
- (A) $\{a^m b^n | m \geq n, n > 0\}$
- (B) $\{a^m b^n | m \geq n, n \geq 0\}$
- (C) $\{a^m b^n | m > n, n \geq 0\}$
- (D) $\{a^m b^n | m > n, n > 0\}$
- $X \rightarrow aX \mid a \Rightarrow L = a^+$
- $Y \rightarrow aYb \mid \epsilon \Rightarrow L = \{a^n b^n \mid n \geq 0\}$
- $S \rightarrow XY \Rightarrow L = \{a^+ a^n b^n \mid n \geq 0\}$
- #a >#b
- $L = \{a^m b^n | m > n, n \geq 0\}$
- Ans: (C) $\{a^m b^n | m > n, n \geq 0$

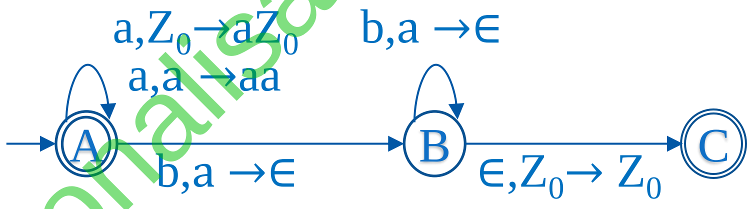
- GATE CS 2017,Set-2,Q40: Consider the following languages:
- $L_1 = \{a^p \mid p \text{ is a prime number}\}$ $L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$
  $L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$ $L_4 = \{a^n b^n \mid n \geq 1\}$
- Which of the following are CORRECT?
- I. L_1 is context-free but not regular. II. L_2 is not context-free.
  III. L_3 is not context-free but recursive. IV. L_4 is deterministic context-free.
- **(A)** I, II and IV only **(B)** II and III only **(C)** I and IV only **(D)** III and IV only
- L_1 is a CSL.
- L_2 is a CFL as only one comparison, which can be done by using PDA.
- L_3 is a CSL as more than one comparison ,So this cannot be done using PDA.
- L_4 is a CFL (as well as DCFL).
- I.False
- II.False
- III.True
- IV.True
- Ans: **(D)** III and IV only

- GATE CS 2018,Q35: Consider the following languages:
- I. $\{a^m b^n c^p d^q \mid m + p = n + q, \text{ where } m, n, p, q \geq 0\}$
  II. $\{a^m b^n c^p d^q \mid m = n \text{ and } p = q, \text{ where } m, n, p, q \geq 0\}$
  III. $\{a^m b^n c^p d^q \mid m = n = p \text{ and } p \neq q, \text{ where } m, n, p, q \geq 0\}$
  IV. $\{a^m b^n c^p d^q \mid mn = p + q, \text{ where } m, n, p, q \geq 0\}$
- Which of the above languages are context-free?
- **(A)** I and IV only **(B)** I and II only **(C)** II and III only **(D)** II and IV only
- I. $m+p = n+q \Rightarrow m-n = q-p \Rightarrow m \geq n$ and $q \geq p$
- Push a's in the stack then we will pop a's after watching b's.Some of a's might left in stack.
- Push c's in the stack and then pop c's by watching d's.
- The remaining a's will be pop by watching some more d's.
- This can be done by using PDA.So its a CFL.
- II. Push a's in stack. Pop a's watching b's.Now push c's in stack.Pop c's watching d's. So it is context free language.
- III. More than one comparison so not CFL
- IV. Comparison in stack can't be done through multiplication .
- So not CFL
- Ans: **(B)** I and II only

- GATE CS 2019,Q31: Which one of the following languages over $\Sigma=\{a,b\}$ is NOT context-free?
- (A)$\{ww^R | w\in\{a,b\}^*\}$
- (B)$\{wa^nb^nw^R | w\in\{a,b\}^*, n\geq 0\}$
- (C)$\{wa^nw^Rb^n | w\in\{a,b\}^*, n\geq 0\}$
- (D)$\{a^nb^i | i\in\{n,3n,5n\}, n\geq 0\}$
- A. CFL [push 'w',pop 'w' for 'w^R']
- B. CFL [push 'w',push 'a^n',pop 'a^n' for 'b^n',pop 'w' for w^R']
- C. cannot be CFL.
- push 'w',push 'a^n' , then 'w^R', now we cannot match 'w^R' with 'w' , because in top of stack we have 'a^n'.
- D. $\{a^nb^n \cup a^nb^{3n} \cup a^nb^{5n}, n\geq 0\}$ CFL
- Ans :(C)$\{wa^nw^Rb^n | w\in\{a,b\}^*, n\geq 0\}$

- GATE CS 2020,Q10:Consider the language L = { a^n | n≥0 }∪{ $a^n b^n$ | n≥0 } and the following statements.
- **I.** L is deterministic context-free.
- **II.** L is context-free but not deterministic context-free.
- **III.** L is not LL(k) for any k.
- Which of the above statements is/are TRUE ?

  (A) I only **(B)** II only **(C)** I and III only **(D)** III only

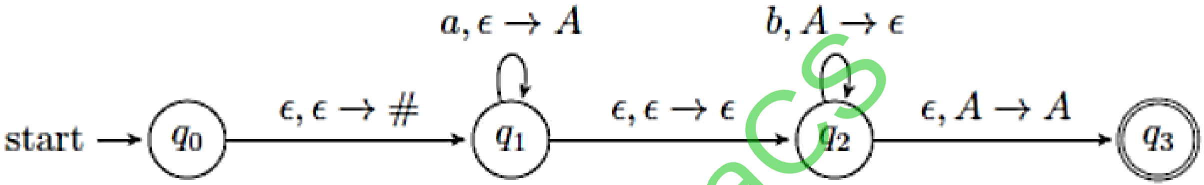


- I.(True) L is DCFL.
- II.(False) L is DCFL so CFL
- III.(True)
- The language is a union of two languages which have common prefixes so not LL(1)
- Ex:{aa, aabb, aaa, aaabbb,….}
- LL(k) parser cannot parse it by using any lookahead ‘k’ symbols.
- Ans: (C) I and III only

- GATE CS 2020,Q32: Consider the following languages.
- $L_1=\{wxyx|w,x,y\in(0+1)^+\}$
- $L_2=\{xy|x,y\in(a+b)^*,|x|=|y|,x\neq y\}$
- Which one of the following is TRUE?
- (A)L_1 is regular and L_2 is context- free.
- (B)L_1 context- free but not regular and L_2 is context-free.
- (C)Neither L_1 nor L_2 is context- free.
- (D)L_1 context- free but L_2 is not context-free.
- $L_1=(0+1)^+0(0+1)^+0+(0+1)^+1(0+1)^+1$
- L_1 is regular as it can represented in regular expression and can be construct finite automata.
- L_2 ={ab,ba,aabb,abba,baab,bbaa,aababa…..}
- $L_2\neq$ {aa,bb,aba,bbb,aaaa,abab,baba,aabaab,abbab,……}
- L_2 is complement of ww and |w|=odd
- We can construct PDA for it.So it is a CFL
- Ans: (A)L_1 is regular and L_2 is context-free

- GATE CS 2021,Set-1,Q1:Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?
- (A) $L_1 \cap L_2$ (B) $L_1 \cdot L_2$ (C) $L_1 - L_2$ (D) $L_1 \cup L_2$
- (A)Regular language is closed under intersection with any language
- $L_1 \cap L_2$ = RL $\cap$ CFL=CFL
- (B)CFL is closed under concatenation, RL$\subseteq$CFL .
- $L_1 \cdot L_2$ = RL.CFL=CFL.CFL=CFL
- (C) CFL is not closed under complement.
- $L_1 - L_2 = L_1 \cap \overline{L_2}$ =RL $\cap$ not necessarily CFL $\neq$CFL
- (D)CFL is closed under union.
- $L_1 \cup L_2$ =RL $\cup$CFL=CFL $\cup$CFL=CFL
- Ans : (C) $L_1 - L_2$

- GATE CS 2021,Set-1,Q51:In a pushdown automaton $P=(Q,\Sigma,\Gamma,\delta,q_0,F)$, a transition of the form, $p \xrightarrow{a,X\to Y} q$ Where $p,q \in Q, a \in \Sigma \cup \{\epsilon\}$, and $X,Y \in \Gamma \cup \{\epsilon\}$, represents $(q,Y) \in \delta(p,a,X)$.Consider the following pushdown automation over the input alphabet $\Sigma=\{a,b\}$ and stack alphabet $\Gamma=\{\#,A\}$.



- The number of strings of length 100 accepted by the above pushdown automaton is _____.
- PDA with final state mechanism.
- {a,aab,aaab,aaaabbb,….}
- $L=\{a^m b^n | m>n, m,n\geq 0\}$
- $|w|=100$
- $\{a^{100}b^0, a^{99}b^1, a^{98}b^2, \ldots. a^{80}b^{20}, \ldots. a^{60}b^{40}, \ldots. a^{51}b^{49}\}$
- Number of string=49-0+1=50
- Ans: 50

- GATE CS 2021,Set-2,Q12: Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?
- (A) $L_1 \cap \overline{L_2}$
- (B) $\overline{\overline{L_1} \cup \overline{L_2}}$
- (C) $L_1 \cup (L_2 \cup \overline{L_2})$
- (D) $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$
- (A) $L_1 \cap \overline{L_2}$=RL ∩ not necessarily CFL= not necessarily CFL
- (B) $\overline{\overline{L_1} \cup \overline{L_2}} = \overline{\overline{L_1}} \cap \overline{\overline{L_2}} = L_1 \cap L_2$ =RL ∩CFL=CFL
- (C) $L_1 \cup (L_2 \cup \overline{L_2})$= RL ∪ Σ^*= Σ^* =RL=CFL
- (D) $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$=(RL ∩CFL) ∪(RL ∩CFL)=CFL
- Ans: (B),(C),(D)

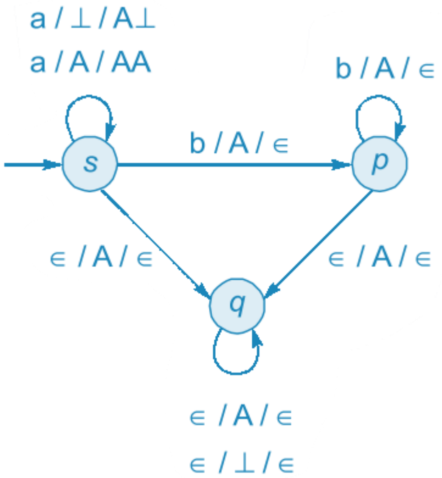
- GATE CS 2021,Set-2,Q41:For a string w , we define w^R to be the reverse of w.For example ,if w=01101 then w^R =10110.which of the following language is/are context free.
- (A) $\{wxw^Rx^R \mid w,x \in \{0,1\}^*\}$
- (B) $\{ww^Rxx^R \mid w,x \in \{0,1\}^*\}$
- (C) $\{wxw^R \mid w,x \in \{0,1\}^*\}$
- (D) $\{wxx^Rw^R \mid w,x \in \{0,1\}^*\}$
- (A) Push for w,push for x,We can't compare w^R with w ,as top of stack will contain x ,Not CFL.
- (B) Push w,pop w for w^R ,push x,pop x for x^R,CFL
- (C) wxw^R =0(0+1)*0+1(0+1)*1 ,RL⇒ CFL
- (D) Push w,push x,pop x for x^R,pop w for w^R.CFL
- $wxx^Rw^R = (wx)(wx)^R$
- Ans : (B),(C),(D)

- GATE CS 2022 | Question: 37 MSQ
- Consider the following languages:
- $L_1=\{a^n w a^n | w \in \{a,b\}^*\}$
- $L_2=\{wxw^R | w,x \in \{a,b\}^*, |w|,|x|>0\}$
- Note that w^R is the reversal of the string w. Which of the following is/are TRUE?
- (A) L_1 and L_2 are regular.
- (B) L_1 and L_2 are context-free.
- (C) L_1 is regular and L_2 is context-free.
- (D) L_1 and L_2 are context-free but not regular.
- $L_1=\{a^n w a^n | w \in \{a,b\}^*\}$ let $n \geq 0$
- If n=0 then (a+b)*,if n=1 then a(a+b)*a for any n>1 $a^n(a+b)^*a^n$
- $L_2=\{wxw^R | w,x \in \{a,b\}^*, |w|,|x|>0\}$ RE :$a(a+b)^+a + b(a+b)^+b$
- L_1=Regular , L_2= Regular
- (A)True.
- (B)RL⊂CFL so True.
- (C) L_1=RL, L_2=RL ⊂CFL,So L_2=CFL True.
- (D)False L_1 and L_2 are RL and CFL so False.
- Ans: A , B , C

- GATE CS 2022 | Question: 38 MSQ
- Consider the following languages:
- $L_1=\{ww|w\in\{a,b\}^*\}$
- $L_2=\{a^nb^nc^m|m,n\geq0\}$
- $L_3=\{a^mb^nc^n|m,n\geq0\}$
- Which of the following statements is/are FALSE?
- (A) L_1 is not context-free but L_2 and L_3 are deterministic context-free.
- (B) Neither L_1 nor L_2 is context-free.
- (C) L_2, L_3 and $L_2\cap L_3$ all are context-free.
- (D) Neither L_1 nor its complement is context-free.
- $L_1=\{ww|w\in\{a,b\}^*\}$is not CFL can be CSL but $\overline{L_1}$=CFL.
- $L_2=\{a^nb^nc^m|m,n\geq0\}$ $a^nb^nc^*$ DCFL.
- $L_3=\{a^mb^nc^n|m,n\geq0\}$ $a^*b^nc^n$ DCFL.
- (A) True.
- (B) False as L_2 is context-free.
- (C) False as $\overline{L_2\cap L_3}= a^nb^nc^n$ not context-free.
- (D)False as $\overline{L_1}$=CFL.
- Ans :B,C,D

- **GATE CS 2023 | Question: 29**
- Consider the context-free grammar G below $S \rightarrow aSb \mid X$
- $X \rightarrow aX \mid Xb \mid a \mid b$,
- Where S and X are non-terminals, and a and b are terminal symbols. The starting non-terminal is S. Which one of the following statements is CORRECT?
- (A) The language generated by G is (a + b)*
- (B) The language generated by G is a*(a + b)b*
- (C) The language generated by G is a*b*(a + b)
- (D) The language generated by G is not a regular language
- $X \rightarrow aX \mid Xb \mid a \mid b$
- {a,b,aa,ab,bb,aaa,aab,abb,bbb,…..} RE=a*(a+b)b*
- $S \rightarrow aSb \mid X$
- {a,b,aa,ab,bb, aaa,aab,abb,bbb,aaab,aabb,….}
- RE=$a^n a^*(a+b)b^*b^n$ +a*(a+b)b*
- a*(a+b)b*
- Ans : (B) The language generated by G is a*(a + b)b*

- **GATE CS 2023 | Question: 30**
- Consider the pushdown automaton (PDA) P below, which runs on the input alphabet {a, b},has stack alphabet { ⊥ , A}, and has three states {s, p, q}, with s being the start state. A transition from state u to state v, labelled c/X/γ , where c is an input symbol or ∈, X is a stack symbol , and γ is a string of stack symbols, represents the fact that in state u, the PDA can read c from the input, with X on the top of its stack, pop X from the stack, push in the string γ on the stack, and go to state v. In the initial configuration, the stack has only the symbol ⊥ in it. The PDA accepts by empty stack.
- Which one of the following options correctly describes the language accepted by P?
- (a) $\{a^m b^n \mid 1 \leq m \text{ and } n < m\}$ (b) $\{a^m b^n \mid 0 \leq n \leq m\}$
- (c) $\{a^m b^n \mid 0 \leq m \text{ and } 0 \leq n\}$ (d) $\{a^m \mid 0 \leq m\} \cup \{b^n \mid 0 \leq n\}$
- L={a,aa,a$^+$,…aab,aaabb,…}
- $\{a^m \mid 1 \leq m\} \cup \{a^m b^n \mid n<m\} = \{a^m b^n \mid 1 \leq m, \text{ and } n < m\}$
- Ans: (a) $\{a^m b^n \mid 1 \leq m \text{ and } n < m\}$



- **GATE CS 2024 | Set 1 | Question: 49**
- Let $G=(V,\Sigma,S,P)$ be a context-free grammar in Chomsky Normal Form with $\Sigma=\{a,b,c\}$ and V containing 10 variable symbols including the start symbol S. The string $w=a^{30}b^{30}c^{30}$ is derivable from S. The number of steps (application of rules) in the derivation $S\rightarrow^{*}w$ is ____.
- The grammar is in CNF if every production in the form V →VV/T
- Number of derivation to generate a string of length n in CNF=2n-1
- |w|=90 ,2*90-1=180-1=179
- Ans :179

- Let w=abc
- S→AX ,X→BC
- A→a ,B →b ,C →c
- Steps 1 S→AX
- Steps 2 S→aX
- Steps 3 S→aBC
- Steps 4 S→abC
- Steps 5 S→abc
- 2*3-1=5

- **GATE CS 2024 | Set 2 | Question: 42**
- Consider a context-free grammar G with the following 3 rules.
- $S \rightarrow aS$, $S \rightarrow aSbS$, $S \rightarrow c$
- Let $w \in L(G)$. Let $n_a(w), n_b(w), n_c(w)$ denote the number of times a, b, c occur in w, respectively. Which of the following statements is/are TRUE?
- (A) $n_a(w) > n_b(w)$ (B) $n_a(w) > n_c(w) - 2$
- (C) $n_c(w) = n_b(w) + 1$ (D) $n_c(w) = n_b(w) * 2$
- L={c , ac , aac , acbc , aacbaac , acbaacbc,…..}
- (A) Not mentioned about c ,Where as c is the minimum string
- acbc number of a's and b's are equal so always not greater than.
- (B) aacbc ,2>2-2 true for any string of language
- (C) True for any string of language
- (D) ac ,1≠0*2 , acbaacbc ,3 ≠ 2*2 .False
- Ans :(B),(C)