

Compiler Design

Chapter 2: Parsing

GATE CS Lectures
by Monalisa

Section 7: Compiler Design($\cong 5$ mark)

- Lexical analysis, parsing, syntax-directed translation. Runtime environments. Intermediate code generation . Local optimization, Data flow analyses: constant propagation, liveness analysis, common subexpression elimination.

- Chapter 1: Introduction to Compiler [Language processing System ,Compiler ,Phases of Compiler , Lexical Analysis]

- Chapter 2: Parsing [Syntax Analysis , CFG, Ambiguous Grammar , Recursive Grammar ,Left Factoring ,Top down parser : LL(1),FIRST & FOLLOW , Bottom up parser : shift-reduce parsing ,LR(0),SLR(1),CLR(1), LALR(1), Operator Precedence grammar]

- Chapter 3: SDT , Code optimization &Runtime environments

Syntax Analysis

- Syntax analysis is the second phase of the compiler. It gets the input from the tokens and generates a syntax tree or parse tree.
- The parsing technique is implemented by CFG.
- **Functions of the parser :**
- 1. It verifies the structure generated by the tokens based on the grammar.
- 2. It constructs the parse tree.
- 3. It reports the errors.
- 4. It performs error recovery.
- Parser can detect errors during construction of syntax tree and grammar of language.
- Ex: such as an arithmetic expression with unbalanced parentheses.
- Parser cannot detect errors such as:
- 1. Variable re-declaration
- 2. Variable initialization before use.
- 3. Data type mismatch for an operation.
- The above issues are handled by Semantic Analysis phase

- **Error-Recovery Strategies**

- 1. Panic mode, 2. Phrase level, 3. Error productions, 4. Global correction

- **Types of parser :**

- There are two types of parsers for grammars: topdown and bottom-up.

- Top-down methods build parse trees from the top (root) to the bottom (leaves), while bottom-up methods start from the leaves and work their way up to the root.

- In either case, the input to the parser is scanned from left to right, one symbol at a time.

- **CFG:** Finite Set of rules which are used to generate the string is called as grammar.

- It has 4 tuples $G=(V,T,P,S)$

- Classification of Grammar

- Grammar can be classified in two ways

- 1. Based on Derivation tree

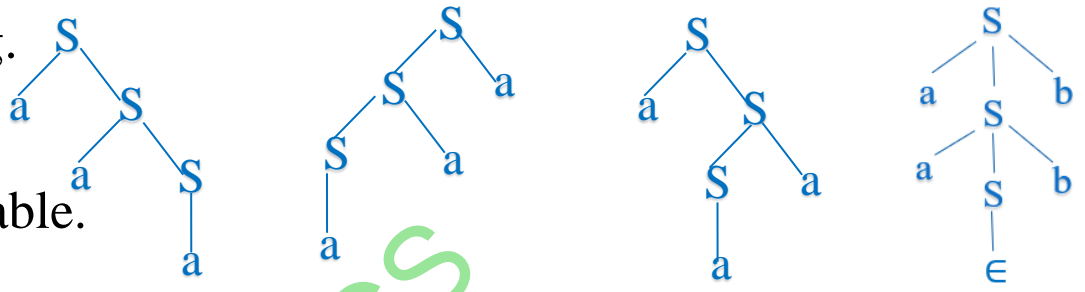
- Ambiguous Grammar
- Unambiguous Grammar

- 2. Based on number of string

- Recursive Grammar
- Non Recursive Grammar

- Ambiguous Grammar: The grammar is said to be ambiguous if more than one parse tree exist for at least one string.

- Ex: $S \rightarrow aS | Sa | a$, $w = aaa$



- Ambiguity of CFG is undecidable.

- Unambiguous Grammar :

- The grammar is said to be unambiguous if there exist unique parse tree for every input string. Ex: $S \rightarrow aSb | \epsilon$, $w = aabb$

- No algorithm exist to convert ambiguous grammar to unambiguous grammar except operator grammar.

- The Ambiguous grammar which can't be converted to unambiguous is called inherent Ambiguous grammar.

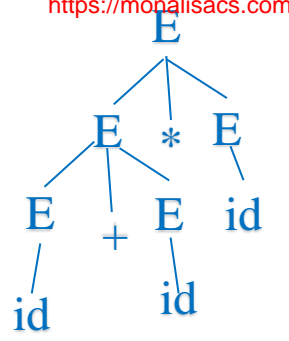
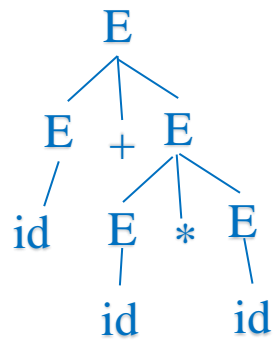
- Operator | Expression grammar can be converted to unambiguous by redefine grammar using associativity & operator precedence.

- Precedence: id, bracket > ^ > *, | > +, -

- ^ is right associative, *, |, +, - are left associative.

Removal of Ambiguity from Expression Grammar

- $E \rightarrow E + E \mid E - E \mid E * E \mid E \wedge E \mid (E) \mid id$
- $W = id + id * id$
- This is a ambiguous grammar.
- In parse tree highest precedence operator is always at lower level than lower precedence.
- It grow left side if operator is left associative & grow right if it is right associative
- Lets rewrite unambiguous grammar



Operator	Associativity	Variable	Grammar
+, -	Left	E	$E \rightarrow E + F \mid E - F \mid F$
*	Left	F	$F \rightarrow F * G \mid G$
^	Right	G	$G \rightarrow H \wedge G \mid H$
(), id		H	$H \rightarrow (E) \mid id$

- Find associativity & operator precedence of all the operator ?
- $S \rightarrow S@W \mid W$
- $W \rightarrow W\#Y \mid Y$
- $Y \rightarrow Y\$A \mid A$
- $A \rightarrow B\%A \mid A\&B \mid id$
- Sol: @ < # < \$ < % , & , id
- Left associative @ , # \$, &
- Right associative %
- **GATE2000-21, ISRO2015-24:** Given the following expression grammar:
- $E \rightarrow E * F \mid F + E \mid F$
- $F \rightarrow F - F \mid id$
- which of the following is true?
- (A) * has higher precedence than +
- (B) – has higher precedence than *
- (C) + and - have same precedence
- (D) + has higher precedence than *
- Ans: (B) – has higher precedence than *

- Recursive Grammar :If at least one production contain same variable both at LHS and RHS. Ex: $S \rightarrow aSb \mid \epsilon$
- Non Recursive Grammar :If no Production contain same variable both at LHS and RHS
- Ex: $S \rightarrow aA \mid b, A \rightarrow a$
- Non Recursive \rightarrow Finite Language
- Recursive \rightarrow Infinite Language
- **Types of Recursion:**
- 1.Left Recursion
- The Grammar is said to be left recursive if left most variable of RHS is same as variable of LHS.
- Ex: $A \rightarrow Aa \mid b$
- 2.Right Recursion
- The Grammar is said to be right recursive if the right most variable of RHS is same as variable of LHS.
- Ex: $A \rightarrow aA \mid b$
- 3.General Recursion
- The Grammar is said to be general recursive if it is neither left nor right recursive . Ex : $A \rightarrow aAb \mid b$

- If the grammar is left recursive then parser may go to infinite loop.
- To avoid looping we need to convert left recursive grammar to right recursive grammar.

Conversion of LRG → RRG:

- $1. A \rightarrow A\alpha/\beta$ $\Rightarrow A \rightarrow \beta A'$
- $\beta\alpha^*$ $\Rightarrow A' \rightarrow \alpha A' / \epsilon$
- $2. A \rightarrow A\alpha_1 / A\alpha_2 | \dots A\alpha_n / \beta$ $\Rightarrow A \rightarrow \beta A'$
- $\beta\alpha^*$ $\Rightarrow A' \rightarrow \alpha_1 A' / \alpha_2 A' | \dots \alpha_n A' / \epsilon$
- $3. A \rightarrow A\alpha / \beta_1 / \beta_2 | \dots \beta_n$ $\Rightarrow A \rightarrow \beta_1 A' / \beta_2 A' | \dots \beta_n A'$
- $\beta\alpha^*$ $\Rightarrow A' \rightarrow \alpha A' / \epsilon$
- $4. A \rightarrow A\alpha_1 / A\alpha_2 | \dots A\alpha_n / \beta_1 / \beta_2 | \dots \beta_n$ $\Rightarrow A \rightarrow \beta_1 A' / \beta_2 A' | \dots \beta_n A'$
- $\beta\alpha^*$ $\Rightarrow A' \rightarrow \alpha_1 A' / \alpha_2 A' | \dots \alpha_n A' / \epsilon$
- Ex 1: $A \rightarrow Aab|c$ $\Rightarrow A \rightarrow cA'$
- $\beta\alpha^*$ $\Rightarrow A' \rightarrow ab A' | \epsilon$
- Ex 2: $S \rightarrow SaS|bS|a$ $\Rightarrow S \rightarrow aS'|bSS'$
- $\beta\alpha^*$ $\Rightarrow S' \rightarrow aS S' | \epsilon$
- Ex 3: $E \rightarrow E+E|E^*E|(E)|id$ $\Rightarrow E \rightarrow idE'|(E)E'$
- $\beta\alpha^*$ $\Rightarrow E' \rightarrow +EE'|*EE'| \epsilon$

- **Grammar with common prefix:**

- If more than one production start with same sequence of grammar symbol then the grammar is called as Grammar with common prefix.

- Ex: $A \rightarrow aAa | aAb | \epsilon$

- Left Factoring:

- Left factoring is a grammar transformation that is useful for top-down parsing.

- The process of removing common prefix or eliminating nondeterminism is called as left factoring.

- $A \rightarrow \alpha\beta_1 | \alpha\beta_2 | \alpha\beta_3$

$$\Rightarrow A \rightarrow \alpha A'$$

$$\Rightarrow A' \rightarrow \beta_1 | \beta_2 | \beta_3$$

- Ex 1: $A \rightarrow ab | ac | ad | ae$

$$\Rightarrow A \rightarrow aB$$

$$\Rightarrow B \rightarrow b | c | d | e$$

- Ex 2: $E \rightarrow E + E | E * E | (E) | id$

$$\Rightarrow E \rightarrow EE' | (E) | id$$

$$\Rightarrow E' \rightarrow +E | *E$$

- Ex 3: $S \rightarrow SaSbS | SbSaS | \epsilon$

$$\Rightarrow S \rightarrow SS' | \epsilon$$

$$\Rightarrow S' \rightarrow aSbS | bSaS$$

- The grammar with both left & right recursive is always ambiguous.

- Left factoring will not remove ambiguity.

● Classification of Parser

- 1) Top down parser
- 2) Bottom up parser

● **Top down Parser:**

- The process of constructing parse tree starting with root & going upto the leaves or children is called top down parsing.
- Top down parser simulate left most derivation.
- It takes the grammar which is free from ambiguity , left recursion & common prefix.
- Top down parser is very slow . Average time complexity $O(n^3)$,n=number of token.

● Types of top-down parsing :

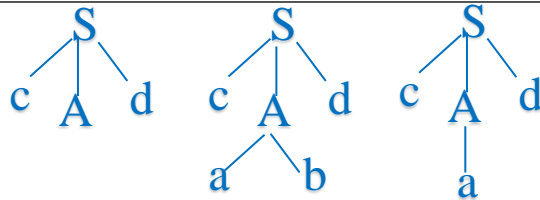
- 1) Recursive descent parsing/Bruteforce Technique [with backtracking]
- 2) Predictive parsing(LL1) [without backtracking]

● **Recursive descent parsing:**

- This parsing method may involve backtracking, that is, making repeated scans of the input.
- Backtracking is costly. Debugging is very difficult.

Ex: Consider the grammar $S \rightarrow cAd$

$A \rightarrow ab|a$



input string $w=cad$.

Step1: Initially create a tree with single node labeled S . An input pointer points to 'c', the first symbol of w .

Step2: The leftmost leaf 'c' matches the first symbol of w , so advance the input pointer to the second symbol of w 'a' and consider the next leaf 'A'.

Expand A using the first alternative.

Step3: The second symbol 'a' of w also matches with second leaf of tree. So advance the input pointer to third symbol of w 'd'.

But the third leaf of tree is 'b' which does not match with the input symbol 'd' Hence discard the chosen production and reset the pointer to second position.

This is called **backtracking**.

Step4: Now try the second alternative for A .

❖ If matching doesn't occur then match with alternative.

If it match at least one alternative then parsing is successful else fail.

Predictive parsing:

- No backtracking.
- Grammar must be free from ambiguity , left recursion & common prefix.

LL(1) Parser:

The first “L” : scanning the input from left to right,the second “L”:producing a leftmost derivation, and the “1” :using one input symbol of look ahead at each step to make parsing action decisions.

The current parsing symbol is called look ahead symbol.

Block Diagram of LL(1) Parser:

LL(1) parser consist of 3 component

- 1) Input Buffer
- 2) Parse stack
- 3) Parse table

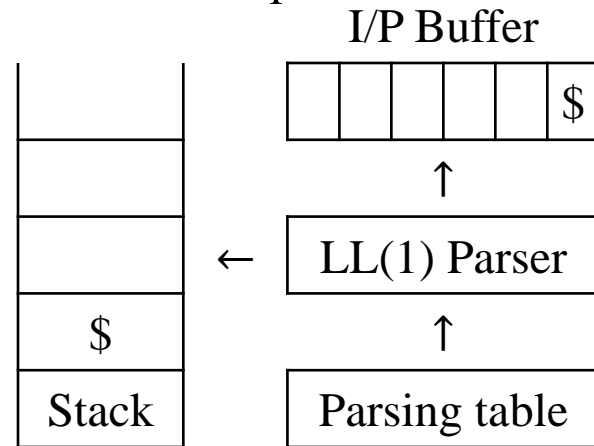
LL(1) Grammar:

The grammar for which LL(1) parser can be constructed is called LL(1) grammar.

The grammar is LL(1) if its parse table is free from multiple entries.

Function used to construct LL(1) parse table

- 1.FIRST(X), 2.FOLLOW(A) $[X \in V^+, A \in V]$



FIRST(X):

- FIRST (x) is set of all terminals that may begin any sentential form or production.
- The first terminal which can be derived from a variable in process of derivation.

Rules for FIRST():

- 1) If X is terminal, then FIRST(X) is {X}.
- 2) If $X \rightarrow \epsilon$ is a production, then add ϵ to FIRST(X).
- 3) If X is non-terminal and $X \rightarrow a\alpha$ is a production then add a to FIRST(X).
- 4) If X is non-terminal and $X \rightarrow Y_1 Y_2 \dots Y_k$ is a production, then place a in FIRST (X) if for some i, a is in FIRST(Y_i), and ϵ is in all of FIRST(Y_1), ..., FIRST(Y_{i-1}); that is, $Y_1 \dots Y_{i-1} \Rightarrow \epsilon$. If ϵ is in FIRST(Y_j) for all $j=1,2,\dots,k$, then add ϵ to FIRST(X).
- 5) If $X \rightarrow Y$ & both are non-terminal then FIRST(X)=FIRST(Y).

➤ Ex 1: $A \rightarrow a|b|\epsilon$

• FIRST(A)={ a,b, ϵ }

➤ Ex 2: $S \rightarrow aSb|bSa|\epsilon$

• FIRST(S)={ a,b, ϵ }

➤ Ex 3: $S \rightarrow aA|bB$

➤ $A \rightarrow aA|b$

➤ $B \rightarrow b|\epsilon$

• FIRST(S) = { a,b }

• FIRST(A) = { a,b }

• FIRST(B) = { b, ϵ }

- Ex 4: $S \rightarrow Aa$
- $A \rightarrow b \mid \epsilon$
 - $FIRST(S) = \{a, b\}$
 - $FIRST(A) = \{b, \epsilon\}$
- Ex 7: $S \rightarrow ABCDE$
- $A \rightarrow a \mid \epsilon$
- $B \rightarrow b \mid \epsilon$
- $C \rightarrow c \mid \epsilon$
- $D \rightarrow d$
- $E \rightarrow e \mid \epsilon$
 - $FIRST(S) = \{a, b, c, d\}$
 - $FIRST(A) = \{a, \epsilon\}$
 - $FIRST(B) = \{b, \epsilon\}$
 - $FIRST(C) = \{c, \epsilon\}$
 - $FIRST(D) = \{d\}$
 - $FIRST(E) = \{e, \epsilon\}$

- Ex 5: $S \rightarrow AB$
- $A \rightarrow a \mid \epsilon$
- $B \rightarrow b \mid c$
 - $FIRST(S) = \{a, b, c\}$
 - $FIRST(A) = \{a, \epsilon\}$
 - $FIRST(B) = \{b, c\}$
- Ex 8: $E \rightarrow TE'$
- $E' \rightarrow +TE' \mid \epsilon$
- $T \rightarrow FT'$
- $T' \rightarrow *FT' \mid \epsilon$
- $F \rightarrow (E) \mid id$
- $FIRST(E) = \{ (, id \}$
 - $FIRST(E') = \{ +, \epsilon \}$
 - $FIRST(T) = \{ (, id \}$
 - $FIRST(T') = \{ *, \epsilon \}$
 - $FIRST(F) = \{ (, id \}$

- Ex 6: $S \rightarrow AB$
- $A \rightarrow aA \mid \epsilon$
- $B \rightarrow bB \mid \epsilon$
 - $FIRST(S) = \{a, b, \epsilon\}$
 - $FIRST(A) = \{a, \epsilon\}$
 - $FIRST(B) = \{b, \epsilon\}$

● FOLLOW(A):

- A terminal which can follow a variable during process of derivation.
- FOLLOW(A) is the set of all terminals that may followed to the right of A in any production or any sentential form.

● Rules for FOLLOW():

- 1) If S is a start symbol, then FOLLOW(S) contains \$.
- 2) If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST(β) except ϵ is placed in FOLLOW(B).
- 3) If there is a production $A \rightarrow \alpha B \beta$ where FIRST(β) contains ϵ , then FOLLOW(B) = FOLLOW(A) \cup FIRST(β) - ϵ .
- 4) If $S \rightarrow \alpha A$ or $S \rightarrow A$ then FOLLOW(A) = FOLLOW(S)

➤ Ex-1: $A \rightarrow a | \epsilon$

➤ Ex-3: $S \rightarrow aA$

● FOLLOW(A) = { \$ }

➤ $A \rightarrow aAb | Sa | \epsilon$

➤ Ex-2: $A \rightarrow A(A) | \epsilon$

● FOLLOW(S) = { \$, a }

● FOLLOW(A) = { \$, (,) }

● FOLLOW(A) = { \$, a, b }

Ex-4:

$S \rightarrow aAB$

$A \rightarrow aAc | \epsilon$

$B \rightarrow bB | a$

FOLLOW(S) = { \$ }

FOLLOW(A) = { a, b, c }

FOLLOW(B) = { \$ }

Ex-6:

$S \rightarrow ABCDE$

$A \rightarrow a | \epsilon$

$B \rightarrow b | \epsilon$

$C \rightarrow c | \epsilon$

$D \rightarrow d$

$E \rightarrow e | \epsilon$

FOLLOW(S) = { \$ }

FOLLOW(A) = { b, c, d }

FOLLOW(B) = { c, d }

FOLLOW(C) = { d }

FOLLOW(D) = { e, \$ }

FOLLOW(E) = { \$ }

Ex-5:

$S \rightarrow AB$

$A \rightarrow aA | \epsilon$

$B \rightarrow bB | \epsilon$

FOLLOW(S) = { \$ }

FOLLOW(A) = { b, \$ }

FOLLOW(B) = { \$ }

Ex-7:

$E \rightarrow TE'$

$E' \rightarrow +TE' | \epsilon$

$T \rightarrow FT'$

$T' \rightarrow *FT' | \epsilon$

$F \rightarrow (E) | id$

FOLLOW(E) = { }, \$ }

FOLLOW(E') = { }, \$ }

FOLLOW(T) = { +,), \$ }

FOLLOW(T') = { +,), \$ }

FOLLOW(F) = { *, +,), \$ }

Construction of predictive / LL(1) Parse Table:

For each production $A \rightarrow \alpha$ of the grammar, do the following:

- 1) For each terminal a in $FIRST(A)$ add $A \rightarrow \alpha$ to $M[A, a]$.
- 2) If ϵ is in $FIRST(A)$, then for each terminal b in $FOLLOW(A)$, add $A \rightarrow \alpha$ to $M[A, b]$.
If ϵ is in $FIRST(A)$ and $\$$ is in $FOLLOW(A)$, add $A \rightarrow \alpha$ to $M[A, \$]$ as well.

The grammar G is LL(1) if predictive parse table is free from multiple entries.

Ex-1: $A \rightarrow aA \mid b$

	FIRST	FOLLOW
A	a, b	\$

	a	b	\$
A	$A \rightarrow aA$	$A \rightarrow b$	

Ex-2:

$S \rightarrow aA \mid Bb$

$A \rightarrow aA \mid b$

$B \rightarrow bB \mid \epsilon$

	FIRST	FOLLOW
S	a, b	\$
A	a, b	\$
B	b, ϵ	b

	a	b	\$
S	$S \rightarrow aA$	$S \rightarrow Bb$	
A	$A \rightarrow aA$	$A \rightarrow b$	
B		$B \rightarrow bB$ $B \rightarrow \epsilon$	

Since there are more than one production, the grammar is not LL(1) grammar.

All ϵ production should be placed FOLLOW of LHS variable.

- Ex-3:
- $S \rightarrow Aa|bB$
- $A \rightarrow aA|c$
- $B \rightarrow bB|\epsilon$

	FIRST	FOLLOW
S	a,b,c	\$
A	a,c	a
B	b, ϵ	\$

	a	b	c	\$
S	$S \rightarrow Aa$	$S \rightarrow bB$	$S \rightarrow Aa$	
A	$A \rightarrow aA$		$A \rightarrow c$	
B		$B \rightarrow bB$		$B \rightarrow \epsilon$

- Ex-4:
- $S \rightarrow Aa|bB$
- $A \rightarrow aA|Bb|d$
- $B \rightarrow SB|b$

	FIRST	FOLLOW
S	a,b,d	\$,a,b,d
A	a,b,d	a
B	a,b,d	b,a,d,\$

	a	b	d	\$
S	$S \rightarrow Aa$	$S \rightarrow bB$ $S \rightarrow Aa$	$S \rightarrow Aa$	
A				
B				

- If any terminal is repeated in FIRST(A) then the grammar is not LL(1)
- Ex-5: $S \rightarrow (S)| \epsilon$

	FIRST	FOLLOW
S	(, ϵ	\$,)

	()	\$
S	$S \rightarrow (S)$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$

- Ex-6:
- $E \rightarrow TE'$
- $E' \rightarrow +TE' | \epsilon$
- $T \rightarrow FT'$
- $T' \rightarrow *FT' | \epsilon$
- $F \rightarrow (E) | id$

	id	()	+	*	\$
E	$E \rightarrow TE'$	$E \rightarrow TE'$				
E'			$E' \rightarrow \epsilon$	$E' \rightarrow +TE'$		$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$	$T \rightarrow FT'$				
T'			$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$	$T' \rightarrow *FT'$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$	$F \rightarrow (E)$				

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	FIRST	FOLLOW
E	(,id), \$
E'	+, ϵ), \$
T	(,id	+,), \$
T'	*, ϵ	+,), \$
F	(,id	+, *,), \$

Short Cut method for testing LL(1) Grammar:

1. If the grammar is free from ϵ production & for every production of the form $A \rightarrow \alpha_1 / \alpha_2 / \alpha_3 | \dots \alpha_n$ the set $\text{FIRST}(\alpha_1) \cap \text{FIRST}(\alpha_2) \dots \text{FIRST}(\alpha_n) = \emptyset$ then grammar is LL(1).

Ex-4

$S \rightarrow Aa|bB$

$A \rightarrow aA|Bb|d$

$B \rightarrow SB|b$

2. If the grammar contain ϵ production & for every production of the form $A \rightarrow \alpha_1 / \alpha_2 / \alpha_3 | \dots \alpha_n$ the set $\text{FIRST}(\alpha_1) \cap \text{FIRST}(\alpha_2) \dots \text{FIRST}(\alpha_n) = \emptyset$ & for every production $A \rightarrow \alpha / \epsilon$ then $\text{FIRST}(\alpha) \cap \text{FOLLOW}(A) = \emptyset$ then grammar is LL(1).

Ex-2

$S \rightarrow aA|Bb$

$A \rightarrow aA|b$

$B \rightarrow bB|\epsilon$

3. Every ambiguous grammar is not LL(1).

4. Every left recursive grammar is not LL(1).

5. Every grammar having common prefix is not LL(1).

- Ex-1: $A \rightarrow ab|bc|d$
- Ex-2: $A \rightarrow ab|ac|d$
- Ex-3: $S \rightarrow aSb | \epsilon$
- Ex-4: $S \rightarrow aSb|bSa|\epsilon$
- Ex-5: $S \rightarrow aA |BbA$
 - $A \rightarrow aA|b$
 - $B \rightarrow bB|\epsilon$
- Ex-6: $S \rightarrow Aa|bB$
 - $A \rightarrow bA|dB|\epsilon$
 - $B \rightarrow aBb|d$
- Ex-7: $S \rightarrow aSbS|bSaS| \epsilon$
- Ex-8: $S \rightarrow aABb$
 - $A \rightarrow c|\epsilon$
 - $B \rightarrow d|\epsilon$

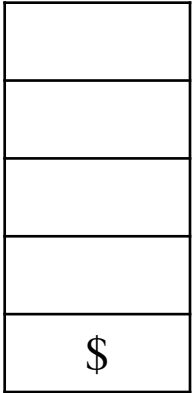
- Ex-1: $\{a\} \cap \{b\} \cap \{d\}$ LL(1)
- Ex-2: $\{a\} \cap \{a\} \cap \{d\}$ not LL(1)
- Ex-3: $\{a\} \cap \{\$,b\}$ LL(1)
- Ex-4: $\{a\} \cap \{b\} \cap \{a,b,\$\}$ not LL(1)
- Ex-5: $\{a\} \cap \{b\}$
 - $\{a\} \cap \{b\}$
 - $\{b\} \cap \{b\}$ Not LL(1)
- Ex-6: $\{a,b,d\} \cap \{b\}$ not LL(1)
- Ex-7: $\{a\} \cap \{b\} \cap \{a,b,\$\}$ not LL(1)
- Ex-8:
 - $\{c\} \cap \{d,b\}$
 - $\{d\} \cap \{b\}$ LL(1)

LL(1) Parsing process using stack:

- 1) Push the start symbol of the grammar into the stack.
 - 2) Compare the topmost symbol of stack to look ahead symbol.
 - 3) If matching occurs ($x=a \neq \$$) then pop off & increment the input pointer
 - 4) If matching doesn't occur ($x \neq a \neq \$$) then perform the push operation again compare the top of the stack with look ahead symbol.
 - 5) After reading the complete string if the stack is empty ($x=a=\$$) then parsing is successful .
- x = top of stack symbol, a =current input symbol, $\$$ =end marker.
 - Ex-1: $S \rightarrow (S) | \epsilon$, $w=(())$

Stack	i/p string	Action
\$	()\$	Push(S)
\$S	()\$	Push($S \rightarrow (S)$)
)S	()\$	Pop
)S	()\$	Push($S \rightarrow (S)$)
)S)	()\$	Pop
)S))\$	Push($S \rightarrow \epsilon$)
)S))\$	Pop
)S))\$	Pop
)S)	\$	accept

	()	\$
S	$S \rightarrow (S)$	$S \rightarrow \epsilon$	$S \rightarrow \epsilon$

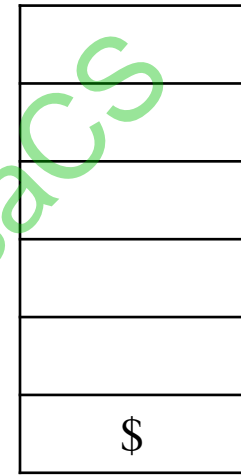


• Number of different push operation =3

- Ex-2:
- $S \rightarrow AA$
- $A \rightarrow aA|b$
- $W=abab$

	a	b	\$
S	$S \rightarrow AA$	$S \rightarrow AA$	
A	$A \rightarrow aA$	$A \rightarrow b$	

Stack	i/p string	Action
\$	abab\$	Push(S)
\$S	abab\$	Push($S \rightarrow AA$)
\$AA	abab\$	Push($A \rightarrow aA$)
\$AAa	abab\$	Pop
\$AA	bab\$	Push($A \rightarrow b$)
\$Ab	bab\$	Pop
\$A	ab\$	Push($A \rightarrow aA$)
\$Aa	ab\$	Pop
\$A	b\$	Push($A \rightarrow b$)
\$b	b\$	Pop
\$	\$	Accept



- Number of different push operation =4
- Minimum number of distinct push operation= n [without ϵ production]
- n =number of tokens
- In case of ϵ Minimum number of distinct push operation= $n-1$

➤ **Bottom up Parser:**

- Constructing a parse tree for an input string beginning at the leaves and going towards the root is called bottom-up parsing.

- Ex: $E \rightarrow E+T | T$, $T \rightarrow T * F | F$, $F \rightarrow (E) | id$

- $W = id * id$

- A general type of bottom-up parser is a shift-reduce parser.

- The class of grammars for which shift-reduce parsers can be built, the LR grammars.

- Bottom up parser simulates reverse of right most derivation.

- Bottom up parser is more powerful than top down parser.

- Average time complexity $O(n^3)$, n =number of token.

- Bottom up parser takes unambiguous grammar for LR parsing.

➤ **Reductions :**

- Bottom-up parsing as the process of "reducing" a string w to the start symbol of grammar.

- At each reduction step, a specific substring matching the body of a production is replaced by the non terminal at the head of that production.

- Ex: A sequence of reductions $id * id$, $F * id$, $T * id$, $T * F$, T , E .

- A reduction is the reverse of a step in a derivation .

- **Handle Pruning:**
- A **handle** of a string is a substring that matches the right side of a production, and whose reduction to the non-terminal on the left side of the production is possible.
- The process of finding & reducing the handle is called as **handle pruning**.

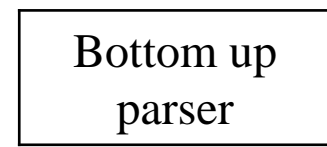
Sentential form	Handle	Reducing Production
$id_1 * id_2$	id_1	$F \rightarrow id$
$F * id_2$	F	$T \rightarrow F$
$T * id_2$	id_2	$F \rightarrow id$
$T * F$	$T * F$	$T \rightarrow T * F$
T	T	$E \rightarrow T$

Grammar:
 $E \rightarrow E + T | T$
 $T \rightarrow T * F | F$
 $F \rightarrow (E) | id$

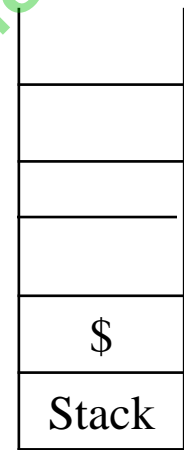
I/P Buffer



↑



↑↓



- **Block diagram of bottom up parser:**
 - It consist of 3 component
- 1) Input buffer
 - 2) Parse stack
 - 3) Parse table

● **Shift-Reduce Parsing:**

● Shift-reduce parsing is a form of bottom-up parsing in which a stack holds grammar symbols and an input buffer holds the rest of the string to be parsed.

● We use \$ to mark the bottom of the stack and also the right end of the input.

● During a left-to-right scan of the input string, the parser shifts zero or more input symbols onto the stack, until it is ready to reduce a string of grammar symbols on top of the stack.

● It then reduces to the head of the appropriate production.

● The parser repeats this cycle until it has detected an error or until the stack contains the start symbol and the input is empty.

● There are four possible actions a shift-reduce parser:

(1) shift, (2) reduce, (3) accept, (4) reject

● **Shift:** Shift the next input symbol onto the top of the stack.

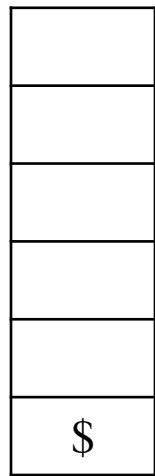
● **Reduce:** The parser replaces the handle within a stack with a variable.

● **Accept:** At the end of parsing if the stack contains only the start symbol then the string is accepted and parsing is successful.

● **Reject:** At the end of parsing if the stack contains anything other than start symbol then the string is reject and parsing is unsuccessful.

● Ex-1:

	Stack	i/p string	Action
● $S \rightarrow AA$	\$	abab\$	Shift
● $A \rightarrow aA b$	\$a	bab\$	Shift
● $W=abab$	\$ab	ab\$	Reduce($A \rightarrow b$)
	\$aA	ab\$	Reduce($A \rightarrow aA$)
	\$A	ab\$	Shift
	\$Aa	b\$	Shift
	\$Aab	\$	Reduce($A \rightarrow b$)
	\$AaA	\$	Reduce($A \rightarrow aA$)
	\$AA	\$	Reduce($S \rightarrow AA$)
	\$S	\$	Accept

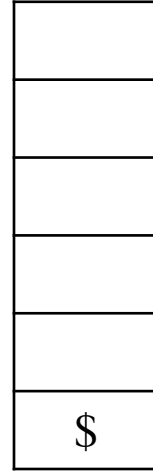


- Number of different reduce operation =3
- Maximum number of reduce moves that can be taken by Shift reduce parser / bottom up parser for a grammar with no ϵ and unit production to parse a string of n token is **n-1**.

● $|abab|=4$

● Number of reduce operation =4-1=3

Ex-2:	Stack	i/p string	Action
$E \rightarrow E+T T$	\$	$id_1 * id_2 \$$	Shift
$T \rightarrow T * F F$	$\$id_1$	$*id_2 \$$	Reduce($F \rightarrow id$)
$F \rightarrow (E) id$	$\$F$	$*id_2 \$$	Reduce($T \rightarrow F$)
$W = id_1 * id_2$	$\$T$	$*id_2 \$$	Shift
	$\$T*$	$id_2 \$$	Shift
	$\$T * id_2$	\$	Reduce($F \rightarrow id$)
	$\$T * F$	\$	Reduce($T \rightarrow T * F$)
	$\$T$	\$	Reduce($E \rightarrow T$)
	$\$E$	\$	accept



- Number of different reduce operation = 4
- Maximum number of reduce moves = $n-1$ [without unit, \in production]
- $|id_1 * id_2| = 3-1 = 2$
- In case of unit production $n-1 + \text{number of unit production}$.
- $2+2[E \rightarrow T, T \rightarrow F] = 4$

Conflicts During Shift-Reduce Parsing

1. Shift-reduce conflict: The parser cannot decide whether to shift or to reduce.

Consider grammar: $E \rightarrow E+E \mid E^*E \mid id$, input :id+id*id

Stack	i/p string	Action	Stack	i/p string	Action
\$E+E	*id\$	Reduce $E \rightarrow E+E$	\$E+E	*id\$	Shift
\$E	*id\$	Shift	\$E+E*	id\$	Shift
\$E*	id\$	Shift	\$E+E*id	\$	Reduce $E \rightarrow id$
\$E*id	\$	Reduce $E \rightarrow id$	\$E+E*E	\$	Reduce $E \rightarrow E^*E$
\$E^*E	\$	Reduce $E \rightarrow E^*E$	\$E+E	\$	Reduce $E \rightarrow E+E$
\$E	\$	Accept	\$E	\$	Accept

2. Reduce-reduce conflict: The parser cannot decide which of several reductions to make. Consider grammar: $M \rightarrow R+R \mid R+c$, $R \rightarrow c$, input :c+c

Stack	i/p string	Action	Stack	i/p string	Action
\$	c+c\$	Shift	\$	c+c\$	Shift
\$c	+c\$	Reduce $R \rightarrow c$	\$c	+c\$	Reduce $R \rightarrow c$
\$R	+c\$	Shift	\$R	+c\$	Shift
\$R+	c\$	Shift	\$R+	c\$	Shift
\$R+c	\$	Reduce $R \rightarrow c$	\$R+c	\$	Reduce $M \rightarrow R+c$
\$R+R	\$	Reduce $M \rightarrow R+R$	\$M	\$	Accept
\$M	\$	Accept			

• Classification of bottom up parser

1) Operator Precedency parser

2) LR Parser

- LR(0) item: LR(0), SLR(1)

- LR(1) item: CLR(1),LALR(1)

• Operator-precedence parsing

- An efficient way of constructing shift-reduce parser is called operator precedence parsing .

- **Operator-grammar**: These grammars have the property that no production on right side is ϵ or has two adjacent non terminals.

- Example: Consider the grammar: $E \rightarrow EAE \mid (E) \mid -E \mid id$

$$A \rightarrow + \mid - \mid * \mid / \mid ^$$

- The right side EAE has three consecutive non-terminals, so not operator grammar.

- The grammar can be written as follows:

- $E \rightarrow E+E \mid E-E \mid E^*E \mid E/E \mid E^E \mid -E \mid id$

- Operator grammar can be ambiguous or unambiguous.

- In operator grammar every terminal is operator.
- Only terminal are used for operator precedence grammar
- Operator grammar work on precedence & associativity property.
- Operator precedence grammar :the operator grammar for which an operator precedence parser can be constructed is called operator grammar.

• **Operator precedence relations:**

- There are three precedence relations namely $< . , = , . >$

- 1) $a < . b$:a yields precedence to b.b reduce before a.
- 2) $a = b$:a has the same precedence as b. reduce according to associativity.
- 3) $a . > b$:a takes precedence over b.a reduce before b.

• **Rules for constructing precedence parse table:**

- Let θ_1 & θ_2 be two operations.

- 1) If θ_1 has higher precedence than θ_2 , then make $\theta_1 . > \theta_2$ and $\theta_2 < . \theta_1$.
- 2) If θ_1 and θ_2 , are of equal precedence, then make $\theta_1 . > \theta_2$ and $\theta_2 . > \theta_1$ if operators are left associative, $\theta_1 < . \theta_2$ and $\theta_2 < . \theta_1$ if right associative.

- 'id' $>$ '^' is right-associative $>$ '*', '/' left-associative and $>$ '+', '-' left-associative $>$ '\$'

Operator precedence parsing algorithm:

Let a is the top of stack & b is the look ahead symbol.

- 1) If $a < b$, or $a = b$ then shift b onto the stack; advance ip to the next input symbol;
- 2) Else if $a . > b$ then /*reduce*/
repeat {pop the stack until the top stack terminal is related by $<$.to the terminal most recently popped}
- 3) If $a=b=\$$ parsing successful .

Stack implementation of operator precedence parsing:

Operator precedence parsing uses a stack and precedence relation table for its implementation of above algorithm.

The initial configuration of an operator precedence parsing is stack $\$$,input $w \$$

Advantages of operator precedence parsing:

- 1) It is easy to implement.
- 2) Once an operator precedence relation is made between all pairs of terminals of a grammar ,the grammar can be ignored.

Disadvantages of operator precedence parsing:

- 1) It is hard to handle tokens like the minus sign (-) which has two different precedence.
- 2) Only a small grammar can be parsed using operator-precedence parser.

- Consider Grammar:
- $E \rightarrow E+E \mid E-E \mid E * E \mid E/E \mid E^E \mid (E) \mid id$
- Input string : $id + id * id$

Stack i/p string Action

\$	<. id+id*id\$	Shift
\$id	.> +id*id\$	Pop
\$	<. +id*id\$	Shift
\$+	<. id*id\$	Shift
\$+id	.> *id\$	Pop
\$+	<. *id\$	Shift
\$+*	<. id\$	Shift
\$+*id	.> \$	Pop
\$+*	.> \$	Pop
\$+	.> \$	Pop
\$	\$	Accept



	+	-	*	/	^	id	()	\$
+	.>	.>	<	<	<	<	<	.>	.>
-	.>	.>	<	<	<	<	<	.>	.>
*	.>	.>	.>	.>	<	<	<	.>	.>
/	.>	.>	.>	.>	<	<	<	.>	.>
^	.>	.>	.>	.>	<	<	<	.>	.>
id	.>	.>	.>	.>	.>			.>	.>
(<	<	<	<	<	<	<	=	
)	.>	.>	.>	.>	.>			.>	.>
\$	<	<	<	<	<	<	<		

● **Introduction to LR Parsing:**

● An efficient bottom-up syntax analysis technique that can be used to parse a large class of CFG is called LR(k) parsing.

● The 'L' is for **left-to-right** scanning of the input, the 'R' for constructing a **rightmost derivation in reverse**, and the 'k' for the number of input symbols .

● **Advantages of LR parsing:**

● It recognizes all programming language constructs for which CFG can be written.

● It is an efficient non-backtracking shift-reduce parsing method.

● It detects a syntactic error as soon as possible.

● A grammar that can be parsed using LR method is a proper superset of a grammar that can be parsed with predictive/LL(1) parser.

● **Drawbacks of LR method:**

● It is too much of work to construct a LR parser by hand for a programming language grammar. A specialized tool, called a LR parser generator, is needed. Example: YACC.

● Types of LR parsing method:

● LR(0),SLR(1)- Simple LR ,Easiest to implement, least powerful.

● CLR(1)- Canonical LR ,Most powerful, most expensive.

● LALR(1)- Look-Ahead LR

● Intermediate in size and cost between SLR & CLR.

Construct LR Parse Table:

1. Obtain the augmented grammar.
2. Construct the canonical collection of LR items.
3. Draw the LR Automata.
4. Construct the parse table from LR Automata.

Augmented grammar:

- If G is a grammar with start symbol S , then G' , the *augmented grammar* for G , is G with a new start symbol S' and production $S' \rightarrow S$.
- The grammar which is obtained by adding 1 more production before start symbol is called as augmented grammar.

LR(0) item:

- An LR parser makes shift-reduce decisions by maintaining states to keep track of where we are in a parse.
- An LR(0) item of a grammar G is a production of G with a dot at some position of the body of RHS.
- $A \rightarrow XYZ$
- LR(0) items $A \rightarrow .XYZ$, $A \rightarrow X.YZ$, $A \rightarrow XY.Z$, $A \rightarrow XYZ.$
- $A \rightarrow X.YZ$ indicates that we have just seen on the input a string derivable from X and next to see a string derivable from YZ .
- Item $A \rightarrow XYZ.$ indicates that we have seen the body XYZ and that it may be time to reduce XYZ to A .

- The production $A \rightarrow \epsilon$ generate only one item , $A \rightarrow \cdot$.

- **Function used to generate LR(0) item**

- 1) Closure(I) [I set of items]
- 2) Goto(I,x) [x grammar symbol]

- **Closure of item sets**

- If I is a set of items for a grammar G, then CLOSURE(I) is the set of items constructed from I by two rules:

- 1) Initially ,add every item in I to CLOSURE(I). $I_0 = \text{CLOSURE}(S' \rightarrow \cdot S)$
- 2) If $A \rightarrow \alpha \cdot B \beta$ is in CLOSURE(I) and $B \rightarrow \gamma$ is a production , then add the item $B \rightarrow \cdot \gamma$ to CLOSURE(I),if it is not already there . apply this rule until no more new items can be added to CLOSURE(I).

- **Goto (I,x)**

- $\text{Goto}(A \rightarrow \alpha \cdot x \beta, x) = (A \rightarrow \alpha x \cdot \beta)$

- **Structure of the LR Parsing Table:**

- The parsing table consist of two functions: 1.ACTION, 2.GOTO.

- ACTION function takes as argument a state I and a terminal or \$.

- ACTION part contains shift & reduce of terminal.

- If x is a terminal & $\text{goto}(I,x) = I_j$ then place S_j in ACTION.

- If parser accepts the input and finishes parsing ,then place **acc** in \$ column of ACTION part.

- If the set I contain a final item then place r_i under all the terminal in ACTION part. $r_i =$ reduce by the production numbered i.
- GOTO function takes as argument a state I and a non terminal and contains only shift operation of non terminal.

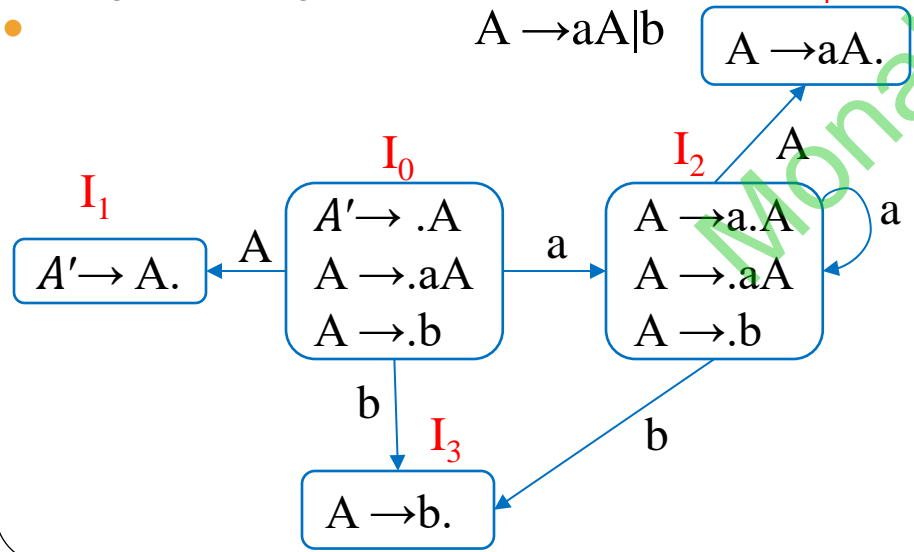
- If x is a non terminal & goto (I,x) = I_j then place j in GOTO.

- **LR(0) grammar:**

- The grammar G is said to LR(0) if its parse table is free from multiple entries.

- Ex 1: $A \rightarrow aA|b$

- augmented grammar $A' \rightarrow A$

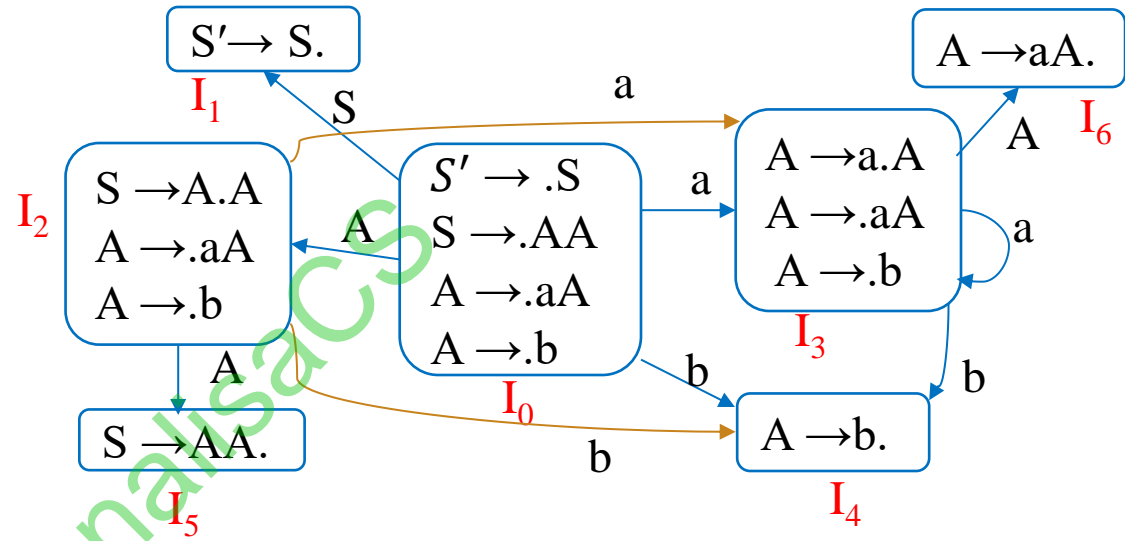


	ACTION			GOTO
	a	b	\$	A
0	S ₂	S ₃		1
1			Acc	
2	S ₂	S ₃		4
3	r ₂	r ₂	r ₂	
4	r ₁	r ₁	r ₁	

- LR(0) Grammar

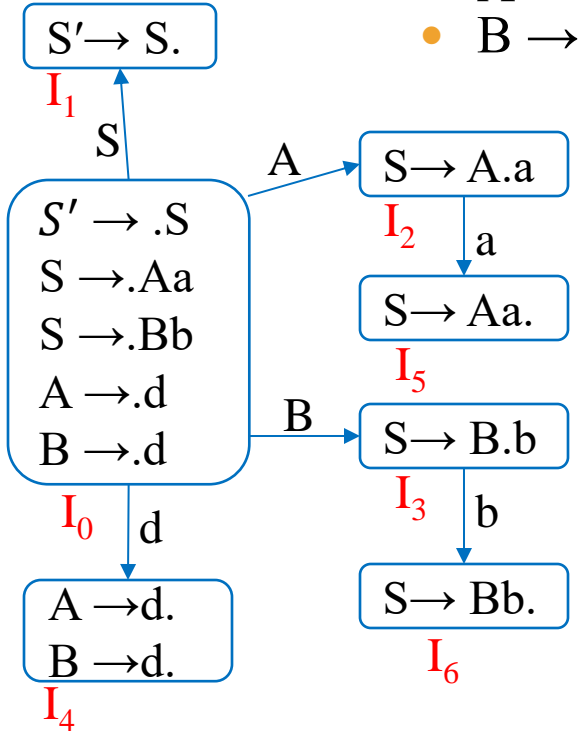
- Ex 2: $S \rightarrow AA$
- $A \rightarrow aA$
- $A \rightarrow b$
- Augmented grammar
- $S' \rightarrow S$
- $S \rightarrow AA$
- $A \rightarrow aA$
- $A \rightarrow b$

ACTION				GOTO	
	a	b	\$	S	A
0	S ₃	S ₄		1	2
1			Acc		
2	S ₃	S ₄			5
3	S ₃	S ₄			6
4	r ₃	r ₃	r ₃		
5	r ₁	r ₁	r ₁		
6	r ₂	r ₂	r ₂		



- LR(0) Grammar

- Ex 3: $S \rightarrow Aa | Bb$
- $A \rightarrow d$
- $B \rightarrow d$
- Augmented grammar
- $S' \rightarrow S$
- $S \rightarrow Aa$
- $S \rightarrow Bb$
- $A \rightarrow d$
- $B \rightarrow d$

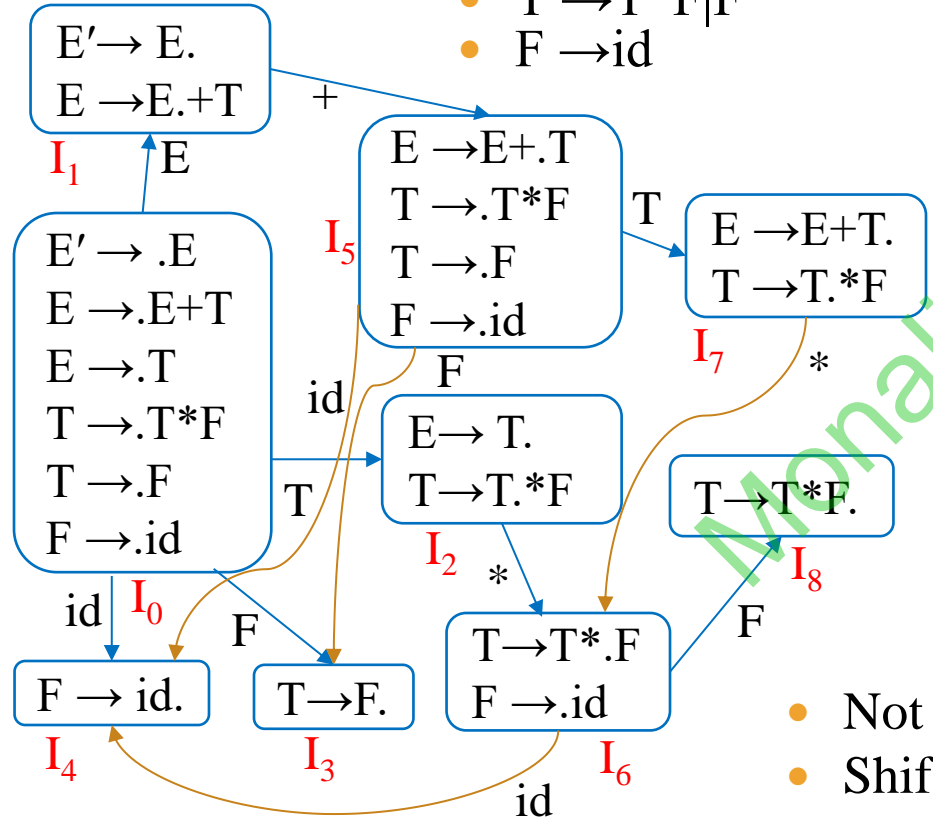


ACTION					GOTO		
	a	b	d	\$	S	A	B
0			S_4		1	2	3
1				Acc			
2	S_5						
3		S_6					
4	r_3/r_4	r_3/r_4	r_3/r_4	r_3/r_4			
5	r_1	r_1	r_1	r_1			
6	r_2	r_2	r_2	r_2			

Monalisacs

- Not LR(0) Grammar.
- Reduce Reduce conflict present.

- Ex 4: $E \rightarrow E+T \mid T$
- $T \rightarrow T * F \mid F$
- $F \rightarrow id$
- Augmented grammar
- $E' \rightarrow E$
- $E \rightarrow E+T \mid T$
- $T \rightarrow T * F \mid F$
- $F \rightarrow id$



ACTION					GOTO		
	id	+	*	\$	E	T	F
0	S ₄				1	2	3
1		S ₅		Acc			
2	r ₂	r ₂	r ₂ /S ₆	r ₂			
3	r ₄	r ₄	r ₄	r ₄			
4	r ₅	r ₅	r ₅	r ₅			
5	S ₄					7	3
6	S ₄						8
7	r ₁	r ₁	r ₁ /S ₆	r ₁			
8	r ₃	r ₃	r ₃	r ₃			

- Not LR(0) Grammar.
- Shift Reduce Conflict present.

LR-parsing algorithm:

- Initially, the parser has 0 on its stack, where 0 is the initial state, and w\$ in the input buffer
- let **a** be the first symbol of w\$;
- while(1)
- { let **s** be the state on top of the stack;
- if (ACTION[s, a] = shift t) {
- push t onto the stack; }
- else if (ACTION[s, a] = reduce (A→β) {
- pop β symbols off the stack;
- let state **t** be on top of the stack;
- push GOTO[t, A] onto the stack; }
- else if (ACTION[s, a] = accept) break;
- else call error-recovery routine;
- }

Stack	Symbols	i/p string	Action
0	\$	id*id\$	Shift to 4
04	\$id	*id\$	Reduce F→id
03	\$F	*id\$	Reduce T→F
02	\$T	*id\$	Shift to 6
026	\$T*	id\$	Shift to 4
0264	\$T*id	\$	Reduce F→id
0268	\$T*F	\$	Reduce T→T*F
02	\$T	\$	Reduce E→T
01	\$E	\$	Accept

	ACTION				GOTO		
	id	+	*	\$	E	T	F
0	S ₄				1	2	3
1		S ₅		Acc			
2	r ₂	r ₂	r ₂ /S ₆	r ₂			
3	r ₄	r ₄	r ₄	r ₄			
4	r ₅	r ₅	r ₅	r ₅			
5	S ₄					7	3
6	S ₄						8
7	r ₁	r ₁	r ₁ /S ₆	r ₁			
8	r ₃	r ₃	r ₃	r ₃			

Conflicts in LR Parsing:

1. Shift-reduce conflict :

- The parser cannot decide whether to shift or to reduce
- If the same state has both shift & reduce option then conflict

2.Reduce-reduce conflict :

- The parser cannot decide which of several reductions to perform
- If the same state contain more than one final item then conflict
- The grammar is LR(0) if & only if it is free from conflict

Viable Prefixes:

- The prefixes of right sentential forms that can appear on the stack of a shift reduce parser are called *viable prefixes*.
- A viable prefix is a prefix of a right-sentential form that does not continue past the right end of the rightmost handle of that sentential form.
- Not all prefixes of right-sentential forms can appear on the stack.
- SLR parsing is based on the fact that LR(0) automata recognize viable prefixes
- Consider grammar : $E \rightarrow E+T \mid T$, $T \rightarrow T*F \mid F$, $F \rightarrow id$
- Examples of Right sentential form : $E \Rightarrow T \Rightarrow T*F \Rightarrow T*id_2 \Rightarrow F*id_2 \Rightarrow id_1*id_2$
- Examples of viable prefix : $id_1, F, T, T*, T*id_2, T*F, E$

Stack	i/p string	Action
\$	$id_1*id_2\$$	Shift
$\$id_1$	$*id_2\$$	Reduce($F \rightarrow id$)
$\$F$	$*id_2\$$	Reduce($T \rightarrow F$)
$\$T$	$*id_2\$$	Shift
$\$T*$	$id_2\$$	Shift
$\$T*id_2$	$\$$	Reduce($F \rightarrow id$)
$\$T*F$	$\$$	Reduce($T \rightarrow T*F$)
$\$T$	$\$$	Reduce($E \rightarrow T$)
$\$E$	$\$$	accept

• **SLR(1) Parser:**

• The SLR method begins with LR(0) items and LR(0) automata.

• **Constructing an SLR-parsing table.**

• **ACTION :**

• (a) If $[A \rightarrow \alpha.a\beta]$ is in I_i and $\text{GOTO}(I_i, a) = I_j$, then set $\text{ACTION}[i, a]$ to “S_j”;
a=terminal.

• (b) If $[A \rightarrow \alpha.]$ is in I_i , then set $\text{ACTION}[i, a]$ to “r_j” for all **a** in $\text{FOLLOW}(A)$; j is reduction number, $A \neq S'$.

• (c) If $[S' \rightarrow S.]$ is in I_i , then set $\text{ACTION}[i, \$]$ to “accept”.

• If any conflicting actions result from the above rules, we say the grammar is not SLR(1).

• **GOTO :**

• The GOTO transitions for state i are constructed for all non terminals A using the rule: If $\text{GOTO}(I_i, A) = I_j$, then $\text{GOTO}[i, A] = j$.

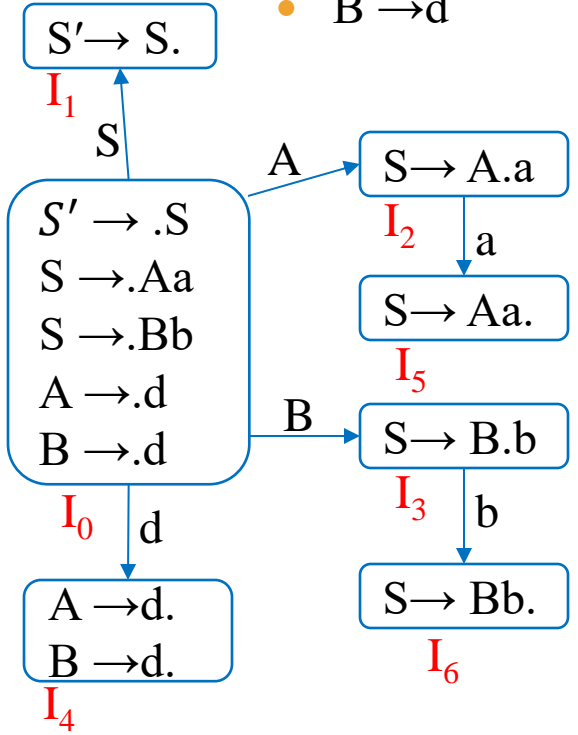
• If SLR(1) parsing table is free from multiple entries then grammar called SLR grammar.

• We usually omit the (1) after the SLR, since we shall not deal here with parsers having more than one symbol of lookahead.

• Every SLR(1) grammar is unambiguous, but there are many unambiguous grammars that are not SLR(1).

- Ex 1:
- $S \rightarrow Aa \mid Bb$
- $A \rightarrow d$
- $B \rightarrow d$

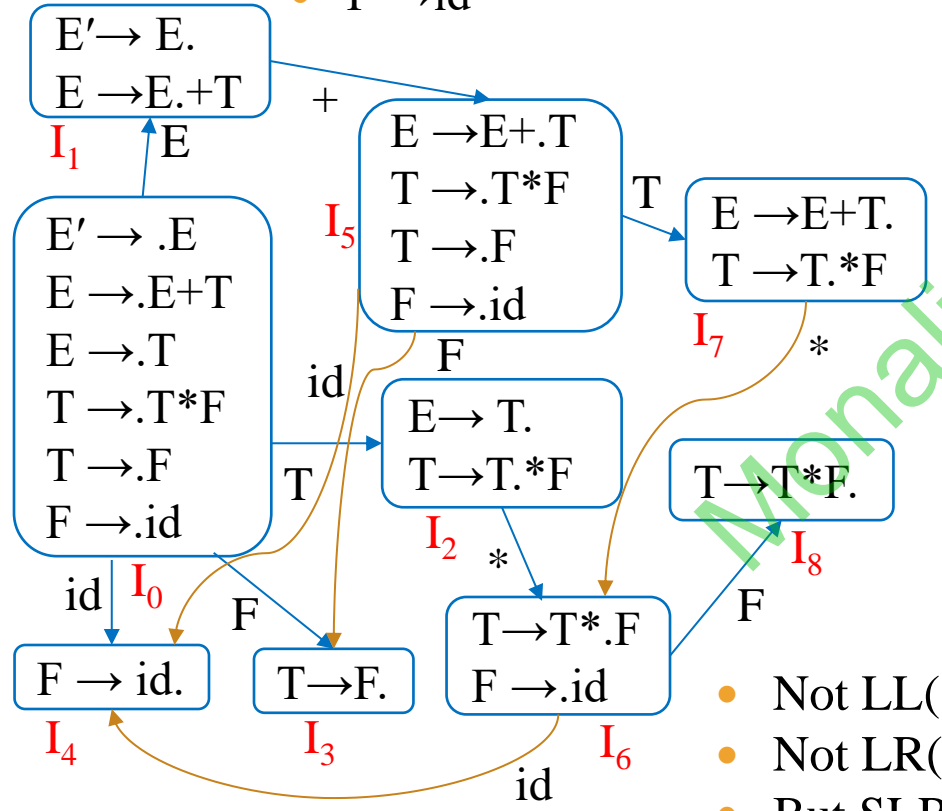
- Augmented grammar
- $S' \rightarrow S$
- $S \rightarrow Aa$
- $S \rightarrow Bb$
- $A \rightarrow d$
- $B \rightarrow d$
- $\text{Follow}(S) = \{\$ \}$
- $\text{Follow}(A) = \{a\}$
- $\text{Follow}(B) = \{b\}$



ACTION					GOTO		
	a	b	d	\$	S	A	B
0			S ₄		1	2	3
1				Acc			
2	S ₅						
3		S ₆					
4	r ₃	r ₄					
5				r ₁			
6				r ₂			

- Not LL(1) Grammar . $\text{First}(Aa) \cap \text{First}(Bb) = d$
- Not LR(0) Grammar . RR conflict present.
- But SLR(1) Grammar

- Ex 2:
- $E \rightarrow E+T \mid T$
- $T \rightarrow T*F \mid F$
- $F \rightarrow id$
- Augmented grammar
- $E' \rightarrow E$
- $E \rightarrow E+T \mid T$
- $T \rightarrow T*F \mid F$
- $F \rightarrow id$
- Follow (E) = { \$, + }
- Follow (T) = { \$, *, + }
- Follow (F) = { \$, *, + }



		ACTION				GOTO		
		id	+	*	\$	E	T	F
0	S ₄					1	2	3
1			S ₅		Acc			
2			r ₂	S ₆	r ₂			
3			r ₄	r ₄	r ₄			
4			r ₅	r ₅	r ₅			
5	S ₄						7	3
6	S ₄							8
7			r ₁	S ₆	r ₁			
8			r ₃	r ₃	r ₃			

- Not LL(1) grammar , Left recursive
- Not LR(0) Grammar , SR Conflict.
- But SLR(1) Grammar

Conflicts in SLR Parsing:

1. Shift-reduce conflict :

- If $\text{follow}(B) \cap x = \emptyset$, SR conflict in LR(0) but not in SLR(1).
- If $\text{follow}(B) \cap x \neq \emptyset$ SR conflict in both LR(0) & SLR(1).

$A \rightarrow \alpha.xB$

$B \rightarrow \gamma.$

$A \rightarrow \alpha.$

$B \rightarrow \gamma.$

2.Reduce-reduce conflict :

- If $\text{follow}(A) \cap \text{follow}(B) = \emptyset$, RR conflict in LR(0) but not in SLR(1).
- If $\text{follow}(A) \cap \text{follow}(B) \neq \emptyset$ RR conflict in both LR(0) & SLR(1)
- The grammar is SLR if & only if it is free from both SR & RR conflict.

Ex 3:

$S \rightarrow AaAb|BbBa$

$A \rightarrow \epsilon$

$B \rightarrow \epsilon$

$S' \rightarrow .S$

$S \rightarrow .AaAb$

$S \rightarrow .BbBa$

$A \rightarrow .$

$B \rightarrow .$

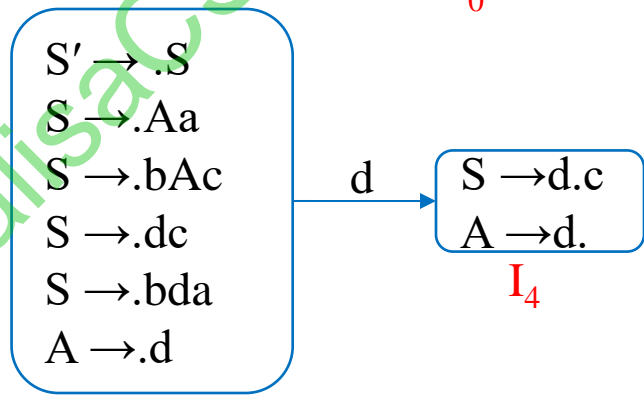
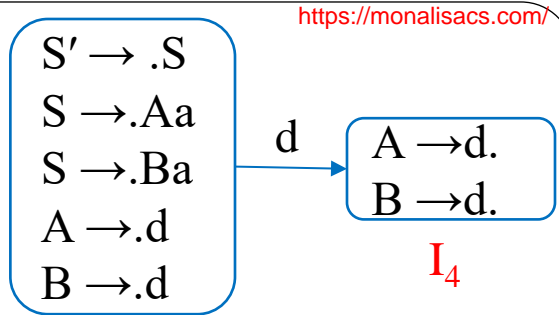
I_0

- Follow (S)={ \$ }
- Follow (A)={ a,b }
- Follow (B)={ a,b }
- Follow(A) \cap Follow(B)={ a,b } $\neq \emptyset$
- RR conflict in both LR(0) & SLR(1)
- Not LR(0) grammar.
- Not SLR(1) grammar.
- LL(1) grammar
- First(AaAb) \cap First(BbBa) = \emptyset

- Ex 4:
- $S \rightarrow Aa|Ba$
- $A \rightarrow d$
- $B \rightarrow d$
- $\text{Follow}(S) = \{\$ \}$
- $\text{Follow}(A) = \{a\}$
- $\text{Follow}(B) = \{a\}$

- $\text{Follow}(A) \cap \text{Follow}(B) = \{a\} \neq \emptyset$
- RR conflict in both LR(0) & SLR(1)
- Not LR(0) grammar .
- Not SLR(1) grammar.
- Not LL(1) grammar
- $\text{First}(Aa) \cap \text{First}(Ba) = \{d\}$

- Ex 5:
- $S \rightarrow Aa|bAc|dc|bda$
- $A \rightarrow d$
- Not LL(1) grammar
- $\text{First}(Aa) \cap \text{First}(dc) = \{d\}$
- $\text{Follow}(A) \cap c = \{c\}$
- SR conflict in both LR(0) & SLR(1)
- Not LR(0) grammar .
- Not SLR(1) grammar.



- SLR(1) is more powerful than LR(0)
- Every LR(0) grammar is SLR(1) but converse not true.
- The number of entries in LR(0) table \geq number of entries in SLR(1) table .
- Both table differ only in ACTION part not GOTO part.
- SLR(1) is more efficient than LR(0).
- **CLR(1) or LR(1) parser:**
- LR(1) =LR(0) +1 Look ahead symbol
- LR(1) determines the reduction dependency on the LA symbol.
- The redundant reduction can be removed hence it is called canonical LR(1).
- **LR(1) item:**
- $[A \rightarrow \alpha. \beta, a], A \rightarrow \alpha\beta$ is a production, β is not ϵ , a is a terminal or right endmarker \$.
- 1 refers to the length of second component called lookahead of the item.
- The lookahead has no effect on item.
- $[A \rightarrow \alpha. , a]$, is a reduction if next input symbol is a .
- The set of a 's will always be a proper subset of FOLLOW(A).

Constructing LR(1) Sets of Items:

The method for building the collection of sets of valid LR(1) items is same as the one for building the canonical collection of sets of LR(0) items.

We need only to modify the two procedures CLOSURE and GOTO.

```

CLOSURE(I) {repeat
  for ( each item  $[A \rightarrow \alpha. B\beta, a]$  in I )
  for ( each production  $B \rightarrow \gamma$  in  $G'$  )
  for ( each terminal b in FIRST( $\beta a$ ) )
    add  $[B \rightarrow. \gamma, b]$  to set I;
until no more items are added to I;
return I; }

GOTO(I, X) {
  for ( each item  $[A \rightarrow \alpha.X\beta, a]$  in I )
    add item  $[A \rightarrow \alpha X. \beta, a]$  to set J ;
return CLOSURE(J ); }

void items( $G'$ ) {
initialize C to {CLOSURE  $[S' \rightarrow. S, \$]$  } ;
repeat
  for ( each set of items I in C )
  for ( each grammar symbol X )
    if ( GOTO(I, X) is not empty and
        not in C )
      add GOTO(I, X) to C;
until no new sets of items are added to C;
}

```

• Canonical LR(1) Parsing Tables:

• Algorithm : Construction of canonical-LR parsing tables.

• ACTION:

• (a) If $[A \rightarrow \alpha. a\beta, b]$ is in I_i and $\text{GOTO}(I_i, a) = I_j$, then set $\text{ACTION}[i, a]$ to “shift j.”

Here a must be a terminal.

• (b) If $[A \rightarrow \alpha. , a]$ is in I_i , $A \neq S'$, then set $\text{ACTION}[i, a]$ to “ r_j ” ; j is reduction number.

• (c) If $[S' \rightarrow S.]$ is in I_i , then set $\text{ACTION}[i, \$]$ to “accept”.

• If any conflicting actions result from the above rules, we say the grammar is not LR(1). The algorithm fails to produce a parser in this case.

• GOTO:

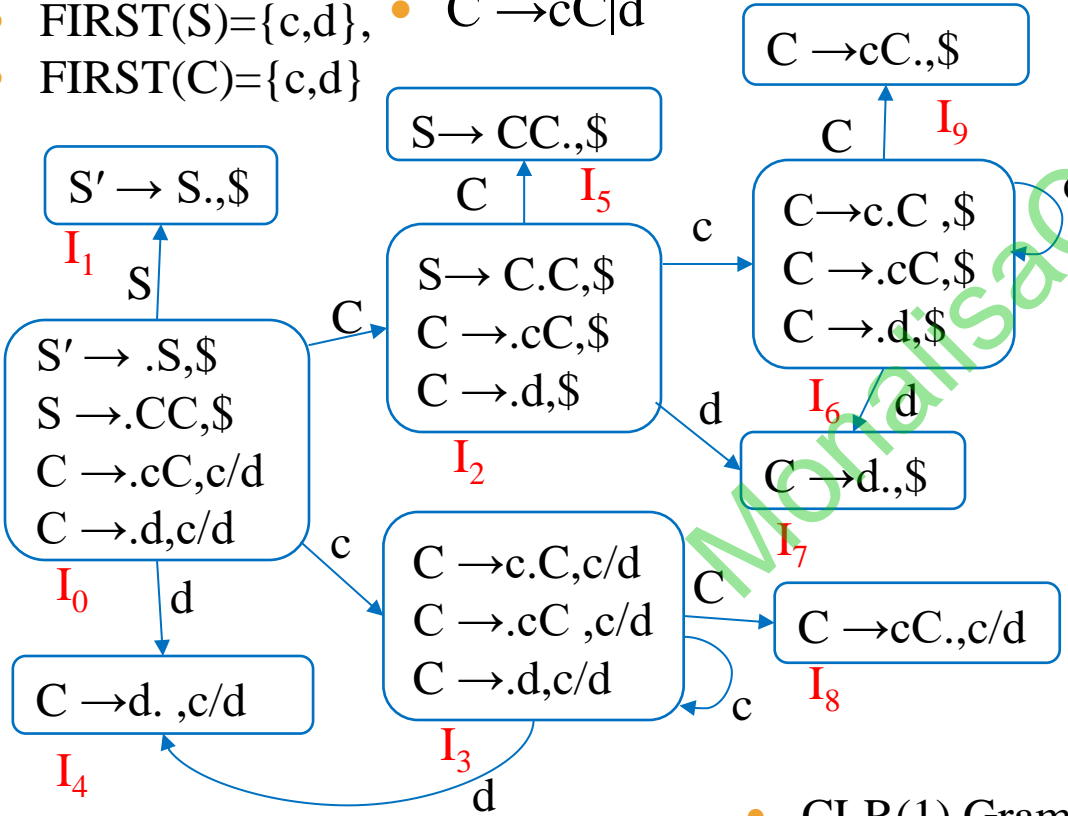
• The GOTO transitions for state i are constructed for all non terminals A using the rule: If $\text{GOTO}(I_i, A) = I_j$, then $\text{GOTO}[i, A] = j$.

• A LR parser using this table is called a CLR(1) parser.

• If the table has no multiple entries then the given grammar is called LR(1) grammar.

- Ex 1:
- $S \rightarrow CC$
- $C \rightarrow cC|d$
- $FIRST(S) = \{c,d\}$,
- $FIRST(C) = \{c,d\}$

- Augmented grammar
- $S' \rightarrow S$
- $S \rightarrow CC$
- $C \rightarrow cC|d$



		ACTION			GOTO	
State	c	d	\$	S	C	
0	S ₃	S ₄		1	2	
1			acc			
2	S ₆	S ₇			5	
3	S ₃	S ₄			8	
4	r ₃	r ₃				
5			r ₁			
6	S ₆	S ₇			9	
7			r ₃			
8	r ₂	r ₂				
9			r ₂			

• CLR(1) Grammar

Conflicts in CLR Parsing:

1. Shift-reduce conflict :

Shift terminal \cap reduction look ahead symbol $\neq \emptyset$, SR conflict in LR(1)

$A \rightarrow \alpha.a\beta, b$
 $B \rightarrow \gamma. , a$

2.Reduce-reduce conflict :

$A \rightarrow \alpha., a$
 $B \rightarrow \gamma., a$

r_i look ahead symbol $\cap r_j$ look ahead symbol $\neq \emptyset$ then its RR conflict in LR(1).

The grammar is LR(1) if & only if it is free from both SR & RR conflict.

Ex 2:

$S \rightarrow Aa|bB$

$A \rightarrow aA|b$

$B \rightarrow b|a$

$S' \rightarrow S., \$$

$S' \rightarrow .S, \$$
 $S \rightarrow .Aa, \$$
 $S \rightarrow .bB, \$$
 $A \rightarrow .aA, a$
 $A \rightarrow .b, a$

$A \rightarrow A.a, \$$

$S \rightarrow b.B, \$$
 $A \rightarrow b., a$
 $B \rightarrow .b, \$$
 $B \rightarrow .a, \$$

$a \cap a = a$ Shift reduce conflict present.

The grammar is not LR(1)

$FOLLOW(A) \cap \{a\} = a$, SR conflict for both LR(0) & SLR

Not LR(0), SLR grammar

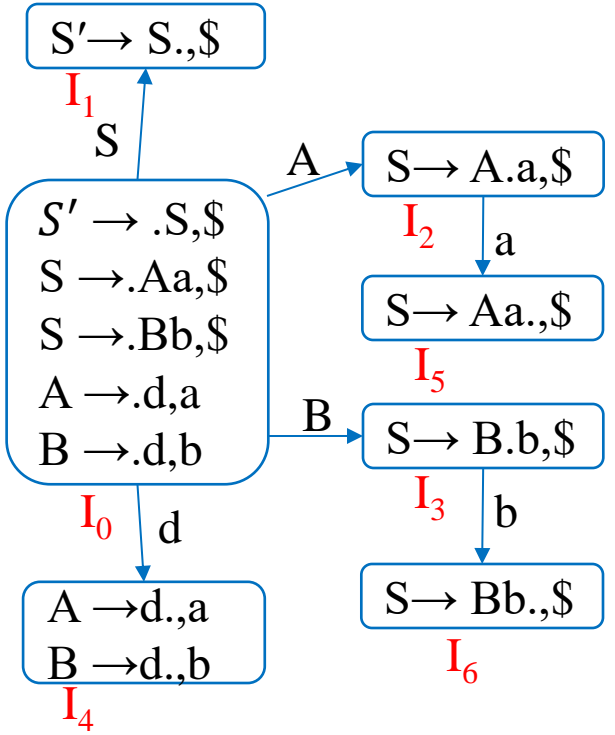
$FIRST(Aa) \cap FIRST(bB) = b$

Not LL(1) grammar

The grammar is not LL(1) or LR as it is ambiguous.

For string "ba" there are more than one parse tree.

- Ex 3:
- $S \rightarrow Aa \mid Bb$
- $A \rightarrow d$
- $B \rightarrow d$



- $a \cap b = \emptyset$, No R-R conflict present for CLR.
- The grammar is LR(1) or CLR
- $FOLLOW(A) \cap FOLLOW(B) = \emptyset$, No R-R conflict for SLR
- The grammar is SLR .
- But for LR(0) R-R conflict present.
- The grammar is not LR(0)
- $FIRST(Aa) \cap FIRST(Bb) = d$
- Not LL(1) grammar

● **LALR(1) Parser:**

● Minimal LR(1) Automata.

● The Automata of CLR(1) parser may contain some states with same production part and different look ahead part.

● Make all these state to single state by union of LA part & again draw the automata & construct the parse table ,which called as LALR table.

● If there are no parsing action conflicts, then the given grammar is said to be an LALR(1) grammar.

● The collection of sets of items are called LALR(1) collection.

● The SLR and LALR tables for a grammar always have the same number of states, and this number is typically several hundred states for a language like C.

● The canonical LR table would typically have several thousand states for the same-size language.

● CLR(1) is more powerful than LALR(1) & LALR(1) is more powerful than SLR(1).

● Every LALR(1) grammar is CLR(1) But every CLR(1) need not be LALR(1).

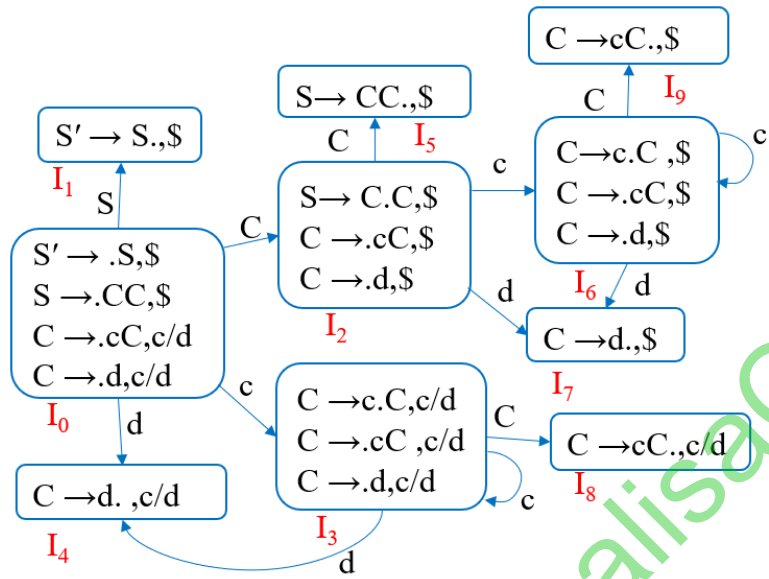
● If CLR(1) have RR conflict or may not have RR conflict ,still LALR(1) may have RR Conflict . LALR(1) have SR conflict if and only if CLR(1) have SR conflict .

● The grammar which is not CLR also not LALR.

● Every SLR(1) grammar is LALR(1) but reverse may not true.

● Number of states in CLR(1) Automata \geq LALR(1) Automata.

- Ex 1:
- $S \rightarrow CC$
- $C \rightarrow cC|d$



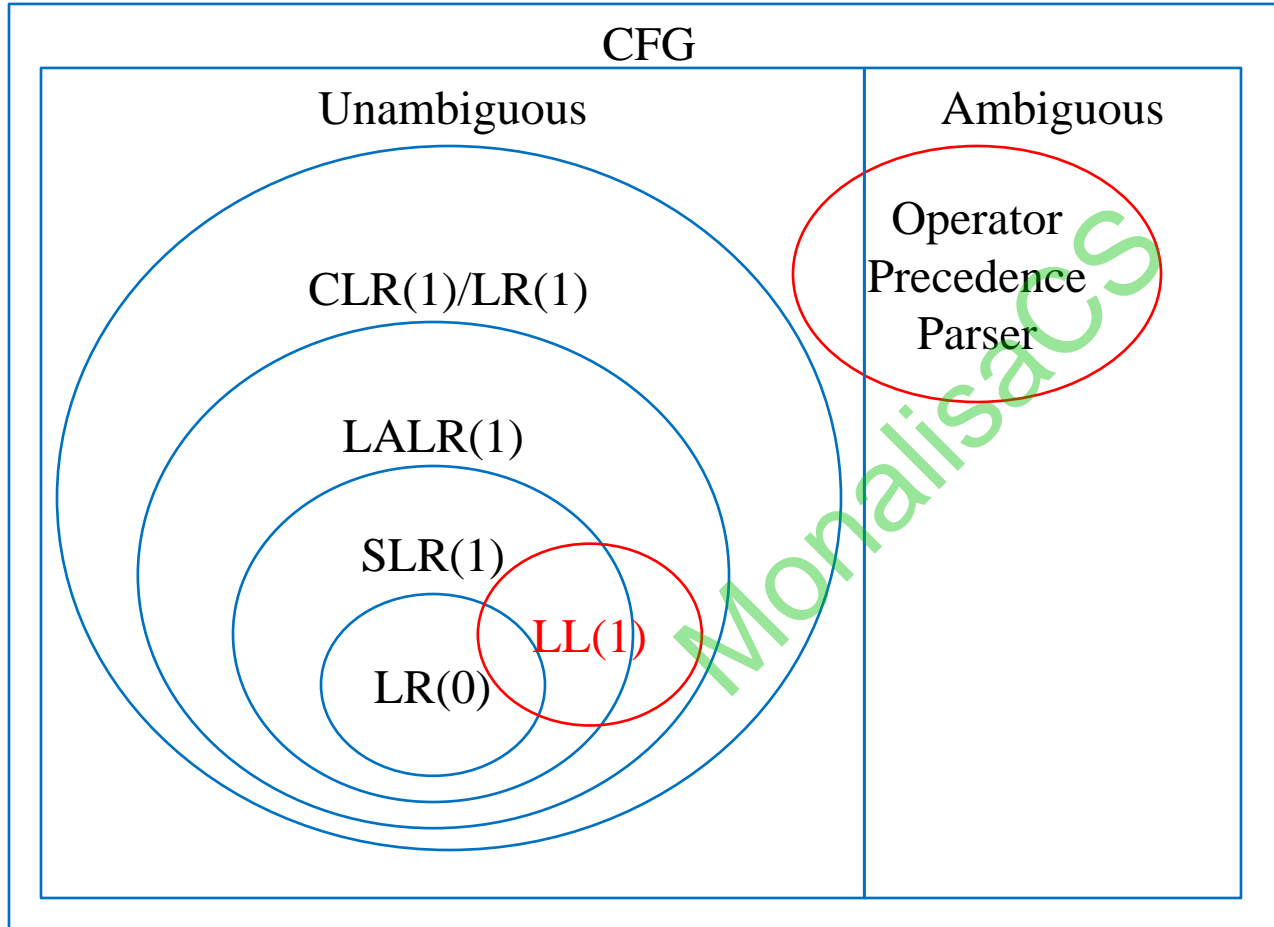
- I_3 and I_6 are replaced by their union.
- I_{36} : $C \rightarrow c.C, c/d/\$$
- $C \rightarrow .cC, c/d/\$$
- $C \rightarrow .d, c/d/\$$
- I_4 and I_7 are replaced by their union.
- I_{47} : $C \rightarrow d., c/d/\$$
- I_8 and I_9 are replaced by their union.
- I_{89} : $C \rightarrow cC., c/d/\$$

		ACTION			GOTO	
State	c	d	\$	S	C	
0	S ₃₆	S ₄₇		1	2	
1			acc			
2	S ₃₆	S ₄₇			5	
36	S ₃₆	S ₄₇			89	
47	r ₃	r ₃	r ₃			
5			r ₁			
89	r ₂	r ₂	r ₂			

- The grammar is LR(0), SLR, CLR(1) & LALR
- LALR parsing table is same as SLR table.
- The grammar is also LL(1).

- $LL(k) \leq LR(k)$
- Set of all $LL(0)$ CFG \subset Set of all $LL(1)$ CFG \subset Set of all $LL(2)$ CFG...
- Set of all $LR(0)$ CFG \subset Set of all $LR(1)$ CFG \subset Set of all $LR(2)$ CFG...
- Set of all $LL(k)$ CFG \subset Set of all $LR(k)$ CFG.
- CLR(1) is more powerful efficient among all the parser.
- But it is very costly hence $LL(1)$ & LALR(1) widely used in the real time compiler construction.
- If one $LL(1)$ grammar having no null production then its also SLR(1)
- Every $LL(1)$ grammar is LALR(1), hence $LR(1)$ as $LALR(1) \subset CLR(1)$
- Power : $LR(0) < SLR(1) < LALR(1) < CLR(1)$
- Easy to implement: $LR(0) > SLR(1) > LALR(1) > CLR(1)$
- Grammar : Set of $LR(0)$ CFG \subset SLR(1) class \subset LALR(1) class \subset CLR(1) Class
- Language : Set of $LR(0)$ Language \subset SLR(1) class \subset LALR(1) class \subset CLR(1) Class
- Parser : $LR(0) < SLR(1) < LALR(1) < CLR(1)$
- Table Size : $LR(0) = SLR(1) = LALR(1) \leq CLR(1)$
- Reduce entry in table: $LR(0) > SLR(1) = LALR(1) > CLR(1)$
- Number of state in Automata : $LR(0) = SLR(1) = LALR(1) \leq CLR(1)$

Relation between all parser



• **GATE CS 2003, Q57:** Consider the grammar shown below.

• $S \rightarrow C C$

• $C \rightarrow c C | d$

• This grammar is

• (A) LL(1)

(B) SLR(1) but not LL(1)

• (C) LALR(1) but not SLR(1)

(D) LR(I) but not LALR(1)

• **Ans: (A) LL(1)**

• **GATE CS 2008, Q55:** An LALR(1) parser for a grammar G can have shift-reduce (S-R) conflicts if and only if

• (A) The SLR(1) parser for G has S-R conflicts

• (B) The LR(1) parser for G has S-R conflicts

• (C) The LR(0) parser for G has S-R conflicts

• (D) The LALR(1) parser for G has reduce-reduce conflicts

• **Ans : (B)**

• **GATE CS 2005, Q60:** Consider the grammar:

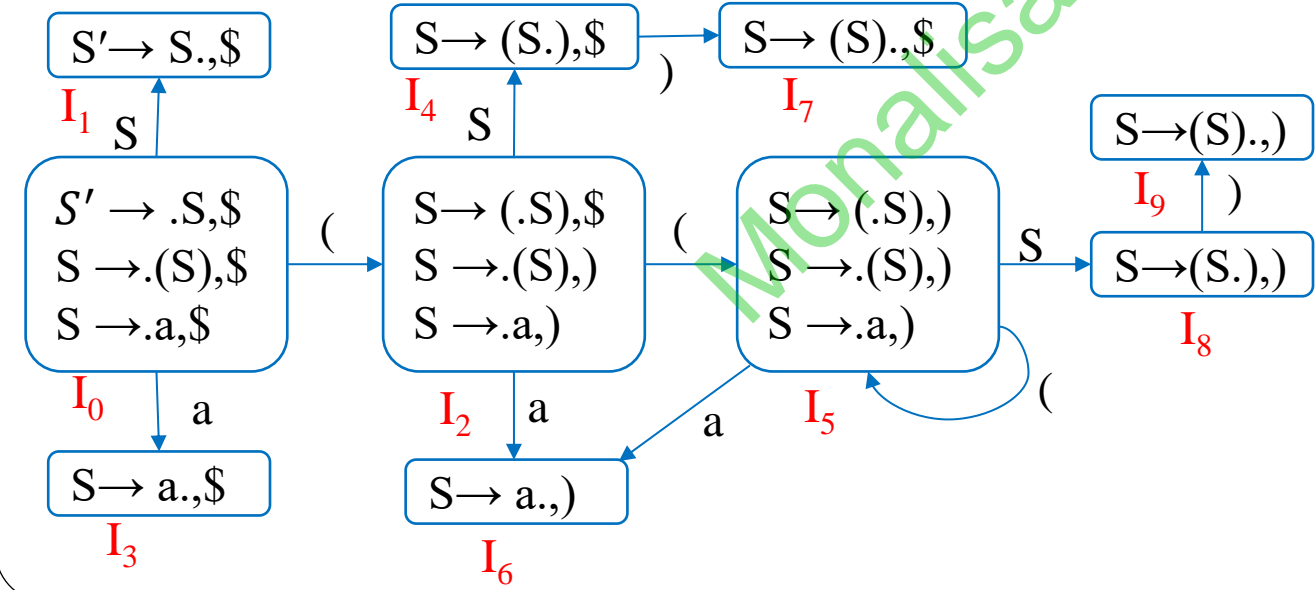
• $S \rightarrow (S) | a$

• Let the number of states in SLR (1), LR(1) and LALR(1) parsers for the grammar be n_1, n_2 and n_3 respectively. The following relationship holds good:

• (A) $n_1 < n_2 < n_3$ (B) $n_1 = n_3 < n_2$ (C) $n_1 = n_2 = n_3$ (D) $n_1 \geq n_3 \geq n_2$

• Number of state in Automata : $LR(0) = SLR(1) = LALR(1) \leq CLR(1)$

• $n_1 = n_3 \leq n_2$



- $n_2 = 10$
- $I_2 \cup I_5 = I_{25}$
- $I_3 \cup I_6 = I_{36}$
- $I_4 \cup I_8 = I_{48}$
- $I_7 \cup I_9 = I_{79}$
- $n_3 = 6, n_1 = 6$
- **Ans:**
(B) $n_1 = n_3 < n_2$