Algorithms
Chapter 3: Decrease and Conquer

GATE CS PYQ
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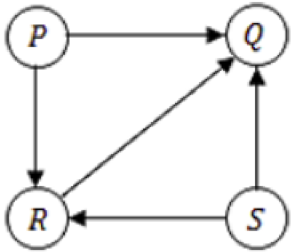
Section 5: Algorithms

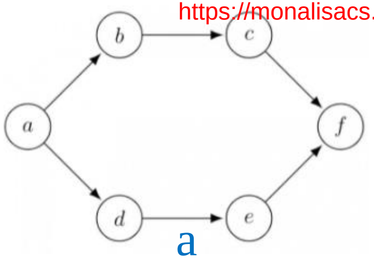
- Searching, sorting, hashing. Asymptotic worst case time and space complexity. Algorithm design techniques : greedy, dynamic programming and divide-and-conquer . Graph traversals, minimum spanning trees, shortest paths
- Chapter 1:Algorithim Analysis:-Algorithm intro , Order of growth ,Asymptotic notation, Time complexity, space complexity, Analysis of Recursive & non recursive program, Master theorem]
- Chapter 2:Brute Force:-Sequential search, Selection Sort and Bubble Sort , Radix sort, Depth first Search and Breadth First Search.
- Chapter 3: Decrease and Conquer :- Insertion Sort, Topological sort,Binary Search .
- Chapter 4: Divide and conquer:-Min max problem , matrix multiplication ,Merge sort ,Quick Sort , Binary Tree Traversals and Related Properties .
- Chapter 5: Transform and conquer:- Heaps and Heap sort, Balanced Search Trees.
- Chapter 6: Greedy Method:-knapsack problem , Job Assignment problem, Optimal merge, Hoffman Coding, minimum spanning trees, Dijkstra's Algorithm.
- Chapter 7: Dynamic Programming:-The Bellman-Ford algorithm ,Warshall's and Floyd's Algorithm ,Rod cutting, Matrix-chain multiplication ,Longest common subsequence ,Optimal binary search trees
- Chapter 8: Hashing.
- Reference : Introduction to Algorithms by Thomas H. Cormen
- Introduction to the Design and Analysis of Algorithms, by Anany Levitin
- My Note

- GATE CS 2014,Set-1,Q13:Consider the directed graph given below. Which one of the following is TRUE?

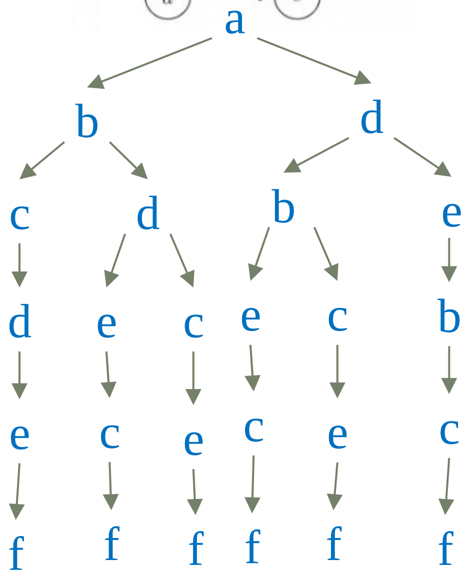


- (A) The graph doesn't have any topological ordering
- (B) Both PQRS and SRPQ are topological ordering
- (C) Both PSRQ and SPRQ are topological ordering
- (D) PSRQ is the only topological ordering
- (A)Wrong , There are no cycles in the graph, so topological orderings exist.
- We can consider P & S as starting vertex, followed by R & Q.
- (B)Wrong
- (C)Both PSRQ & SPRQ are the topological orderings , True.
- (D)Wrong ,multiple topological ordering exist.
- Ans :(C) Both PSRQ and SPRQ are topological ordering

- GATE CS 2016 Set 1 | Q11:
- Consider the following directed graph:



- The number of different topological orderings of the vertices of the graph is ______________.
- Source-removal algorithm
- Indegree :(a,0),(b,1),(c,1),(d,1),(e,1),(f,2) remove a.
- Indegree :(b,0), (d,0),(c,1),(e,1),(f,2) remove b or d, 2 options . let b
- Indegree :(d,0),(c,0),(e,1),(f,2) remove c or d, 2 options . let d
- Indegree :(c,0),(e,0),(f,2) remove c or e, 2 options . let c
- Indegree :(e,0),(f,1) remove e .
- Indegree :(f,0) remove f
- Topological sort1: a , b , d , c , e , f ,TS2:a,b,c,d,e,f
- TS3:a,b,d,e,c,f ,TS4:a,d,b,e,c,f ,TS5:a,d,b,c,e,f ,TS6:a,d,e,b,c,f
- Or
- Start with a and end with f. a_ _ _ _f
- Blank spaces are to be filled with b, c, d, e such that b comes before c, and d comes before e.
- Number of ways to arrange will be = 4!/(2!∗2!)=6
- Ans : 6



- **GATE CS 2017 Set 1 | Question: 48**
- Let *A* be an array of 31 numbers consisting of a sequence of 0's followed by a sequence of 1's. The problem is to find the smallest index *i* such that *A*[*i*] is 1 by probing the minimum number of locations in *A*. The *worst case* number of probes performed by an *optimal* algorithm is ________.
- Worst case example are 01111…. , 0000……1
- It's a sorted sequence 0's fallowed by 1's.
- For sorted sequence we can use binary search.
- At each stage compare 1 with $(l+r)/2$ if its 0 search at right ,if its 1 search at left.
- Searching time is $\Theta(\log n)$.
- *Worst case* number of probes performed by an *optimal* algorithm is $\log_2 31=5$
- Ans : 5

- **GATE CS 2021 Set 1 | Q 9:**
- Consider the following array.

23	**32**	**45**	**69**	**72**	**73**	**89**	**97**

- Which algorithm out of the following options uses the least number of comparisons (among the array elements) to sort the above array in ascending order?
- (A)Selection sort (B)Mergesort
- (C)Insertion sort (D)Quicksort using the last element as pivot
- **Selection sort** :No matter how the data is arranged, there would always be comparisons so the time complexity for best , average and worst case is :$O(n^2)$.
- **Merge Sort** :the best, average and worst case time complexity is:O(nlogn)
- **Insertion Sort :**When elements are already sorted ,The time complexity is :O(n) Best case.
- average and worst case time complexity is:$O(n^2)$
- **Quick Sort :**
  If we use the last or first element as pivot, then Quick Sort will give worst case performance .
- Quick Sort worst case time complexity is $O(n^2)$
- The best and average case time complexity is :O(nlogn)
- Ans : (C)Insertion sort

- **GATE CS 2021 Set 2 | Question: 8**
- What is the worst-case number of arithmetic operations performed by recursive binary search on a sorted array of size *n*?
- A.$\Theta(\sqrt{n})$ B.$\Theta(\log_2 n)$ C.$\Theta(n^2)$ D.$\Theta(n)$
- Recurrence relation for $C_{worst}(n)$: $C_{worst}(n) = C_{worst}\lfloor n/2\rfloor + 1$ for $n > 1$, $C_{worst}(1) = 1$.
- T(n)=T(n/2)+1 T(1)=1
- a=1,b=2,f(n)=1
- $n^{\log_b a} = n^{\log_2 1} = n^0 = 1$
- *Case 2 :* $f(n)=\Theta(n^{\log_b a})$ *then* $T(n)$ *is* $\Theta(n^{\log_b a} * \log_2 n)$
- $T(n)$ *is* $\Theta(\log_2 n)$
- Ans : B.$\Theta(\log_2 n)$

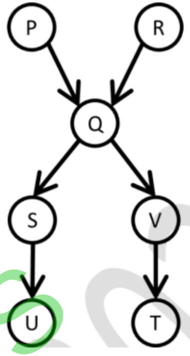
- **GATE DA 2024 | Question: 30**
- Let $F(n)$ denote the maximum number of comparisons made while searching for an entry in a sorted array of size n using binary search.Which **ONE** of the following options is **TRUE**?
- (A) $F(n) = F(\lfloor n/2 \rfloor) + 1$ (B) $F(n) = F(\lfloor n/2 \rfloor) + F(\lceil n/2 \rceil)$
- (C) $F(n) = F(\lfloor n/2 \rfloor)$ (D) $F(n) = F(n - 1) + 1$
- $C_{worst}(n) = C_{worst}\lfloor n/2 \rfloor + 1$ for $n > 1$, $C_{worst}(1) = 1$
- Ans : (A) $F(n) = F(\lfloor n/2 \rfloor) + 1$

- **GATE DA 2024 | Question: 35**
- Consider the following sorting algorithms:
- (i) Bubble sort (ii) Insertion sort (iii) Selection sort
- Which **ONE** among the following choices of sorting algorithms sorts the numbers in the array [4, 3, 2, 1, 5] in increasing order after **exactly two** passes over the array?
- (A) (i) only (B) (iii) only (C) (i) and (iii) only (D) (ii) and (iii) only
- Array Given : [4, 3, 2, 1, 5]

	(i)Bubble sort	(ii) Insertion sort	(iii) Selection sort
1^{st} pass	[3,2,1,4,5]	[3,4,2,1,5]	[1,3,2,4,5]
2^{nd} pass	[2,1,3,4,5]	[2,3,4,1,5]	[1,2,3,4,5]

- Selection sort sorts the array in increasing order after exactly two passes
- Ans : (B) (iii) only

GATE DA 2024 | Question: 41



- Consider the directed acyclic graph (DAG) below:
- Which of the following is/are valid vertex orderings that can be obtained from a topological sort of the DAG?
- (A) P Q R S T U V (B) P R Q V S U T
- (C) P Q R S V U T (D) P R Q S V T U
- Source-removal algorithm :Repeatedly, identify in a remaining digraph a ***source***,which is a vertex with no incoming edges, and delete it along with all the edges outgoing from it.
- (P,0),(R,0),(S,1),(U,1),(V,1),(T,1),(Q,2) Remove P=>TS:P
- (R,0),(S,1),(U,1),(V,1),(T,1),(Q,1) Remove R=>TS:P,R
- (Q,0),(S,1),(U,1),(V,1),(T,1) Remove Q=>TS:P,R,Q
- (S,0),(V,0),(U,1),(T,1) Remove S=>TS:P,R,Q,S Remove V=>TS:P,R,Q,V
- (V,0),(U,0),(T,1) Remove V=>TS:P,R,Q,S,V (S,0),(T,0),(U,1), Remove S=>TS:P,R,Q,V,S
- (U,0),(T,0) Remove T=>TS:P,R,Q,S,V,T (T,0),(U,0) Remove U=>TS:P,R,Q,V,S,U
- (U,0), Remove U=>TS:P,R,Q,S,V,T,U (T,0) Remove T=>TS:P,R,Q,V,S,U,T
- Ans: (B),(D)