Discrete Mathematics
Chapter 3 : Graph Theory

GATE CS PYQ
by Monalisa

- **Section1: Engineering Mathematics**
- **Discrete Mathematics**: Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations , generating functions.
- **Linear Algebra**: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.
- **Calculus**: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.
- **Probability and Statistics**: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

- **Chapter 3 : Graph Theory**
- 3.1 Graphs and Graph Models (17)
- 3.2 Graph Terminology and Special Types of Graphs (21)
- 3.3 Representing Graphs and Graph Isomorphism (22 ,15)
- 3.4 Connectivity (22 ,21,14)dfs & topological
- 3.5 Euler and Hamilton Paths(22,19)
- 3.6 Planar Graphs (21,14,12,11,15)
- 3.7 Graph Coloring(23,22,20,18,16)

- **GATE CS 2010 | Question: 1**
- Let G=(V,E) be a graph . Define $\xi(G)=\sum_d i_d * d$,where i_d is the number of vertices of degree d in G . If S and T are two different trees $\xi(S) = \xi(T)$,then
- (A) |S|=2|T| (B) |S|=|T|-1 (C) |S|=|T| (D) |S|=|T|+1
- Sum of degrees in a graph =2|E|,
- when sum of degrees are same, number of edges must also be same.
- Trees with equal no of edges have equal no of vertices as No of Edges = No of vertices −1, in a tree.
- |S|=|T|
- Ans : (C) |S|=|T|

- **GATE CS 2010 | Question: 28**
- The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph?
- I.7,6,5,4,4,3,2,1 II.6,6,6,6,3,3,2,2 III.7,6,6,4,4,3,2,2 IV.8,7,7,6,4,2,1,1
- (A) I and II (B)III and IV (C)IV only (D)II and IV
- Apply Havel–Hakimi Theorem

- I. 7,6,5,4,4,3,2,1
- 5,4,3,3,2,1,0
- 3,2,2,1,0,0
- 1,1,0,0,0
- 0,0,0,0
- Simple graph

- II. 6,6,6,6,3,3,2,2
- 5,5,5,2,2,2,1
- 4,4,1,1,1,1
- 3,1,0,0,0
- Not Simple graph

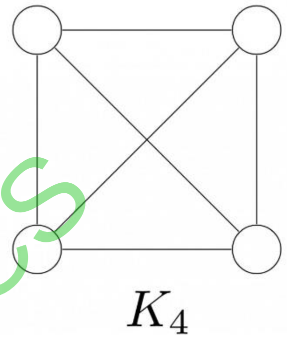
- III.7,6,6,4,4,3,2,2
- 5,5,3,3,2,1,1
- 4,2,2,1,1,0
- 1,1,0,0,0
- 0,0,0,0
- Simple graph

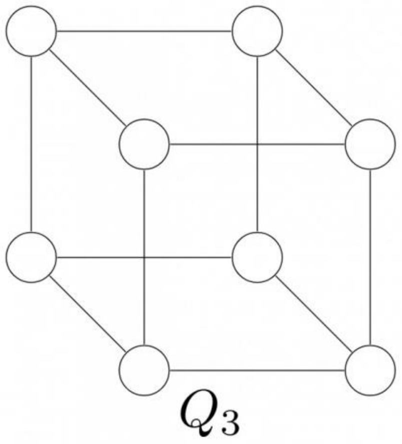
- IV.8,7,7,6,4,2,1,1
- 6,6,5,3,1,0,0
- Not Simple graph

- Ans : (D)II and IV

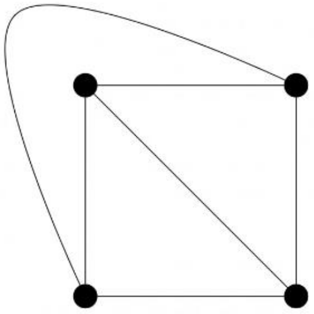
GATE CS 2011 | Question: 17

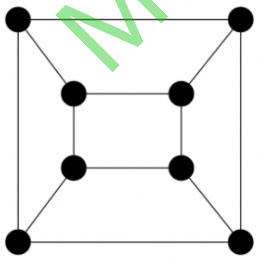
- Which one of the following statements is **TRUE** in relation to these graphs?
- A. K_4 is a planar while Q_3 is not
- B. Both K_4 and Q_3 are planar
- C. Q_3 is planar while K_4 is not
- D. Neither K_4 nor Q_3 is planar





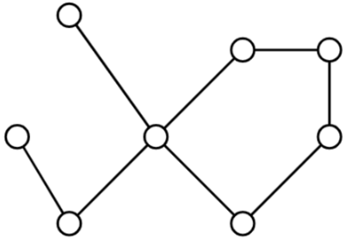
- Both are Planar graphs
- Both graphs can be drawn on a plane without having any crossed edges.
- Ans : B. Both K_4 and Q_3 are planar





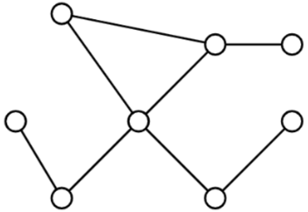
- **GATE CS 2012 | Question: 17**
- Let G be a simple undirected planar graph on 10 vertices with 15 edges. If G is a connected graph, then the number of **bounded** faces in any embedding of G on the plane is equal to
- A.3 B.4 C.5 D.6
- |V|=10 , |E|=15 , |F|=?
- For any planar graph |V|+|F|=|E|+2
- 10+|F|=15+2
- |F|=17-10=7
- Number of bounded face=7-1=6
- Ans : D.6

- ***GATE CS 2012 | Question: 26**
- Which of the following graphs is isomorphic to

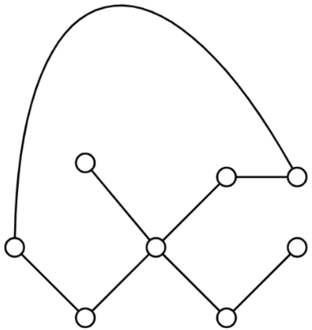


- (A) (B) (C) (D)

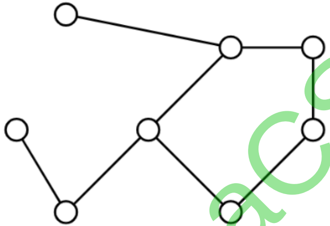
(A)



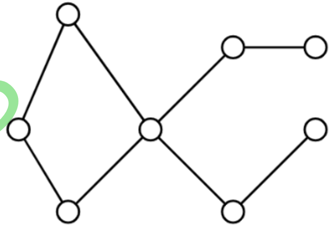
(B)



(C)



(D)



- All graph have 8 vertices & 8 edges
- Degree sequence=1,1,2,2,2,2,2,4
- (A) have a 3-length cycle so not isomorphic .
- (B) have a 5-length cycle ,Degree sequence=1,1,2,2,2,2,2,4 same hence isomorphic .
- (C) have a 5-length cycle ,Degree sequence=1,1,2,2,2,2,3,3 are not same hence not isomorphic
- (D) have a 4-length cycle so not isomorphic .
- Ans : (B)

- ***GATE CS 2012 | Question: 38**
- Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to
- (A)15 (B)45 (C)90 (D)360
- There can be total ${}^{6}C_{4}$ ways to pick 4 vertices from 6. ${}^{6}C_{4}$ = 15.
- There can be 6 different cycle with 4 vertices. For example, consider 4 vertices as a, b, c and d.
- (a, b, c, d,a) , (a, b, d, c,a) , (a, c, b, d,a) , (a, c, d, b,a) , (a, d, b, c,a) , (a, d, c, b,a)
- But (a, b, c, d,a) and (a, d, c, b,a) , (a, b, d, c,a) and (a, c, d, b,a) ,(a, c, b, d,a) and (a, d, b, c,a) are same cycles.
- Three distinct cycles
- So total number of distinct cycles is (15*3) = 45.
- Ans : (B)45

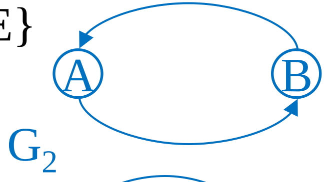
- ***GATE CS 2013 | Question: 24**
- Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is 1/2. What is the expected number of unordered cycles of length three?
- (A)18 (B)1 (C)7 (D)8
- A cycle of length 3 requires 3 vertices.
- Number of ways in which we can choose 3 vertices from 8 = $^{8}C_{3}$=56.
- Probability that 3 vertices form a cycle = 1/2×1/2×1/2=1/8
- Expected number of unordered cycles of length 3=56×1/8=7
- Ans : (C)7

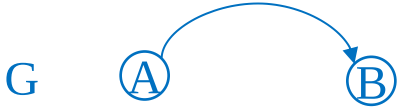
- ***GATE CS 2013 | Question: 25**
- Which of the following statements is/are TRUE for undirected graphs?
- P: Number of odd degree vertices is even.
- Q: Sum of degrees of all vertices is even.
- (A) P only (B) Q only (C) Both P and Q (D) Neither P nor Q
- P:True , Number of odd degree vertices is even.
- Sum of odd deg +sum of even deg =2* number of edges
- Sum of odd deg =2*E-sum of even deg
- Q: True , Sum of degrees of all vertices is even.
- Sum of degrees=2* number of edges
- Ans : (C) Both P and Q

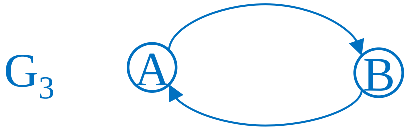
- **GATE CS 2013 | Question: 26**
- The line graph L(G) of a simple graph G is defined as follows:
- There is exactly one vertex v(e) in L(G) for each edge e in G.
- For any two edges e and e′ in G, L(G) has an edge between v(e) and v(e′), if and only if e and e′ are incident with the same vertex in G.
- Which of the following statements is/are TRUE?
- (P) The line graph of a cycle is a cycle. (Q) The line graph of a clique is a clique.
- (R) The line graph of a planar graph is planar. (S) The line graph of a tree is a tree.
- (A) P only (B) P and R only (C) R only (D) P,Q and S only
- (P) True ,every vertices will be

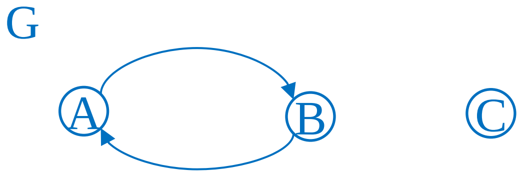
*GATE CS 2014 Set 1 | Question: 3

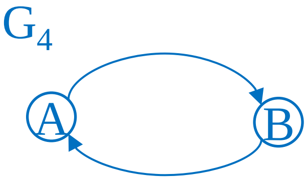
- Let G = (V , E) be a directed graph where V is the set of vertices and E the set of edges. Then which one of the following graphs has the same strongly connected components as G ?
- (A) $G_1 = (V,E_1)$ where $E_1=\{(u,v)|(u,v) \notin E\}$
- (B) $G_2 = (V,E_2)$ where $E_2=\{(u,v)|(v,u) \in E\}$
- (C) $G_3 = (V,E_3)$ where $E_3=\{(u,v)|$ there is a path of length ≤ 2 from u to v in E$\}$
- (D) $G_4 = (V_4,E)$ where V_4 is the set of vertices in G which are not isolated
- **(A)** $G \neq G_1, G_1$ is complement of G .False
- **(B)** $G = G_2 =$ SCC ,change of direction will not affect SCC.True
- **(C)** $G \neq G_3$,Adding more edges will change SCC.False
- **(D)** $G \neq G_4$,removing isolated vertices will decrease number of SCC . False
- Ans : (B) $G_2 = (V,E_2)$ where $E_2=\{(u,v)|(v,u) \in E\}$



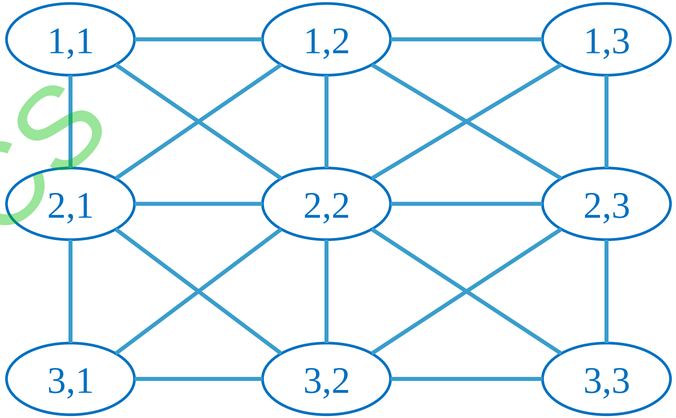








- ***GATE CS 2014 Set 1 | Question: 51**
- Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i,j) \mid 1 \le i \le 12, 1 \le j \le 12\}$.There is an edge between (a,b) and (c,d) if $|a-c| \le 1$ and $|b-d| \le 1$. The number of edges in this graph is______.
- Let's understand with small graph $3x3$ vertices .



- 2*E= 3*4 + 5*4 +8*1=40
- E=20
- A graph with $12x12$ vertices
- 2*E=3*4+5*40+8*100=1012
- E=1012/2=506
- Ans : 506

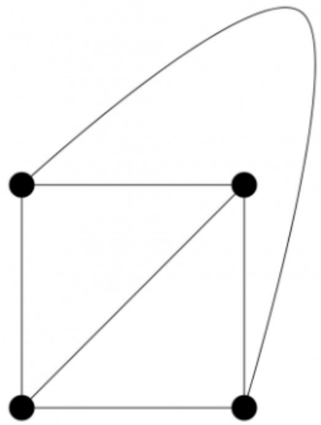
- ***GATE CS 2014 Set 1 | Question: 52**
- An ordered n−tuple $(d_1, d_2, \ldots, d_n)$ with $d_1 \geq d_2 \geq \ldots \geq d_n$ is called *graphic* if there exists a simple undirected graph with n vertices having degrees $d_1, d_2, \ldots, d_n$ respectively. Which one of the following 6-tuples is NOT graphic?
- (A) (1,1,1,1,1,1) (B) (2,2,2,2,2,2) (C) (3,3,3,1,0,0) (D) (3,2,1,1,1,0)
- (A) graphic
- (B) graphic
- (C) (3,3,3,1,0,0) Apply Havel-Hakimi Algorithm
- (2 , 2 ,0 ,0 ,0)
- Not graphic
- (D) (3,2,1,1,1,0) Apply Havel-Hakimi Algorithm
- (1 , 0 , 0 , 1 ,0) =(1 , 1 , 0 , 0 , 0)
- (0 , 0 , 0 , 0) graphic
- Ans : (C) (3,3,3,1,0,0)

- ***GATE CS 2014 Set 2 | Question: 3**
- The maximum number of edges in a bipartite graph on 12 vertices is____
- Maximum number of edges possible in a bipartite graph with n vertices= $\left\lfloor \frac{n^2}{4} \right\rfloor$
- $\left\lfloor \frac{12^2}{4} \right\rfloor$
- =144/4 =36
- Ans : 36

- ***GATE CS 2014 Set 2 | Question: 51**
- A cycle on *n* vertices is isomorphic to its complement. The value of *n* is _____.
- A cycle with n vertices has n edges .
- $|E| = n$,$|E'| = n$
- $|E|+|E'| = \frac{n(n-1)}{2}$
- $n+n = \frac{n(n-1)}{2}$
- $2n = \frac{n(n-1)}{2}$
- $4 = \frac{n(n-1)}{n}$
- n-1=4
- n=5
- Ans : 5

- ***GATE CS 2014 Set 3 | Question: 51**
- If **G** is the forest with ***n*** vertices and ***k*** connected components, how many edges does **G** have?
- (A) $\left\lfloor \frac{n}{k} \right\rfloor$ (B) $\left\lceil \frac{n}{k} \right\rceil$ (C) n-k (D) n-k+1
- A forest is a collection of trees.
- Vertices = n and components =k.
- A component is itself a tree.
- Since there are k components means that every component has a root , k roots.
- In a tree with n vertices and 1 root have n-1 edges .
- In a tree with n vertices and k roots have n-k edges
- Ans : (C) n-k

- ***GATE CS 2014 Set 3 | Question: 52**
- Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on *n* vertices with $\delta \geq 3$, which one of the following is **TRUE**?
- A. In any planar embedding, the number of faces is at least $\frac{n}{2}+2$
- B. In any planar embedding, the number of faces is less than $\frac{n}{2}+2$
- C. There is a planar embedding in which the number of faces is less than $\frac{n}{2}+2$
- D. There is a planar embedding in which the number of faces is at most $\frac{n}{\delta+1}$
- Every vertices have deg ≥ 3
- $3n \leq 2e$
- $3n/2 \leq e$
- Euler's formula $n+f-2 = e$
- $n+f-2 \geq 3n/2$
- $f \geq 3n/2 - n + 2 = n/2+2$
- Ans : A. In any planar embedding, the number of faces is at least $\frac{n}{2}+2$



- ***GATE CS 2015 Set 1 | Question: 54**
- Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is________________.
- By Euler's Formula $f + v = e + 2$
- v=10
- 3*f = 2*e
- f = 2*e / 3
- 2*e / 3 +v =e + 2
- 2e/3-e+10=2
- 10-e/3=2
- e/3=8
- e=24
- Ans : 24

- ***GATE CS 2015 Set 2 | Question: 28**
- A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on *n* vertices, *n* is
- A. A multiple of 4 B. Even
- C. Odd D. Congruent to 0 mod 4, or, 1 mod 4.
- E(G) + E (G')= $\frac{n(n-1)}{2}$
- E(G) = E (G')= $\frac{n(n-1)}{4}$
- Ans : D. Congruent to 0 mod 4, or, 1 mod 4.

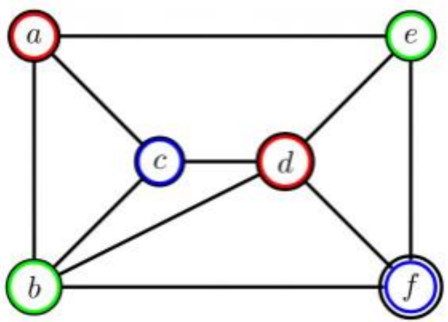
- ***GATE CS 2015 Set 2 | Question: 50**
- In a connected graph, a bridge is an edge whose removal disconnects the graph. Which one of the following statements is true?
- A. A tree has no bridges
- B. A bridge cannot be part of a simple cycle
- C. Every edge of a clique with size ≥ 3 is a bridge (A clique is any complete subgraph of a graph)
- D. A graph with bridges cannot have cycle
- (A) Every edge of a tree is a bridge .False
- (B) A bridge cannot be a part of a single cycle because in a cycle every vertex will be connected with every other vertex in 2 ways. Even if you remove 1 way by deleting an edge still the other way will make sure that the graph is connected . True
- (C) A Clique will never have a bridge because though we remove 1 edge between any 2 vertices those 2 vertices will still be connected with the remaining $(n-2)$ vertices using an edge each.False
- (D) A graph with bridges can have cycle ,False
- Ans : (B)

- ***GATE CS 2016 Set 2 | Question: 03**
- The minimum number of colours that is sufficient to vertex-colour any planar graph is _________.
- **THE FOUR-COLOR THEOREM** The chromatic number of a planar graph is no greater than four.
- Ans : 4

- *GATE CS 2017 Set 1 | Question: 20
- Let T be a tree with 10 vertices. The sum of the degrees of all the vertices in T is ________
- In tree with 10 vertices have 9 edges
- The sum of the degrees of all the vertices in T is =2*9=18
- Ans : 18

- ***GATE CS 2017 Set 2 | Question: 23**
- **G** is an undirected graph with *n* vertices and 25 edges such that each vertex of *G* has degree at least 3. Then the maximum possible value of *n* is __________ .
- $3*n \leq 2*25$
- $n \leq 2*25/3 = 50/3 = 16.67$
- $n = 16$
- Ans :16

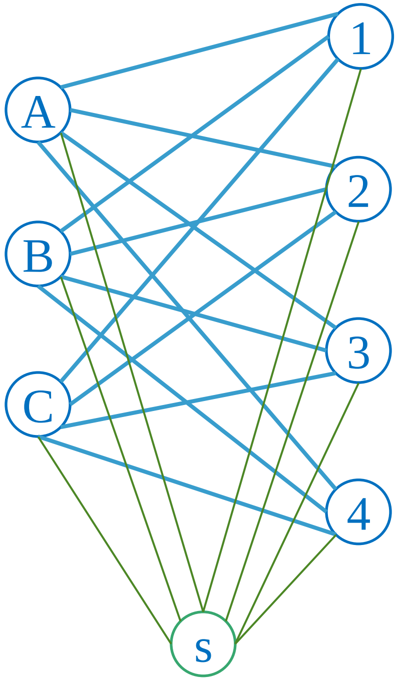
- ***GATE CS 2018 | Question: 18**
- The chromatic number of the following graph is _____
- Ans: 3



- ***GATE CS 2018 | Question: 43**
- Let G be a graph with 100! vertices, with each vertex labelled by a distinct permutation of the numbers 1,2,…,100. There is an edge between vertices ***u*** and ***v*** if and only if the label of ***u*** can be obtained by swapping two adjacent numbers in the label of ***v***. Let ***y*** denote the degree of a vertex in ***G***, and ***z*** denote the number of connected components in ***G***.
  Then, ***y***+10***z***= ______.
- y=? the degree of every vertex(which will be same for all vertices) and z=?number of connected components.
- Let's take a graph with 3! Vertices
- Here {1,2,3,} will be connected with {2,1,3} ,{1,3,2}
- Here we got "2" because we can choose any 2 pairs of adjacent numbers. So, with n, we have n−1 adjacent pairs to swap.
- So, degree will be n−1.
- In our question, degree will be 100−1=99 , y=99
- Number of connected components = 1. z=1
- y=99 and z=1
- ∴y+10z=99+10∗1=109.
- Ans :109

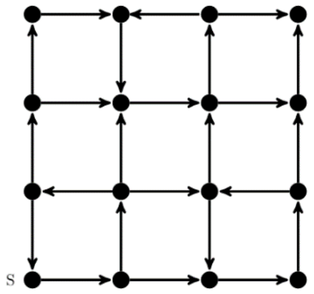
- ***GATE CS 2019 | Question: 12**
- Let G be an undirected complete graph on *n* vertices, where *n*>2. Then, the number of different Hamiltonian cycles in G is equal to
- A.n! B.(n−1)! C.1 D.(n−1)!/2
- A simple circuit in a graph G that passes through every vertex exactly once is called a Hamiltonian circuit/cycle.
- In an undirected complete graph on n vertices, n permutations are possible to visit every node.
- But from these permutations, there are:
- n different places (i.e., nodes) you can start;
- 2 (clockwise or anticlockwise) different directions you can travel.
- So any one of these n! cycles is in a set of 2n cycles which all contain the same set of edges.
- So there are,= (n)! / (2n) = (n−1)! / 2 distinct Hamilton cycles.
- If the graph is unlabeled number of different Hamiltonian cycles become 1
- Since the question does not mention whether the graph is labeled or not,
- Ans: (C) or (D)

- ***GATE CS 2020 | Question: 52**
- Graph G is obtained by adding vertex s to $K_{3,4}$ and making *s* adjacent to every vertex of $K_{3,4}$. The minimum number of colours required to edge-colour G is _______ .
- s has 7 adjacent .
- All 7 edges will have different colours.
- That's why minimum number of colours required to edge-colour=7
- Ans : 7



- ***GATE CS 2021 Set 1 | Question: 16**
- In an undirected connected planar graph G, there are eight vertices and five faces. The number of edges in G is __________.
- $|V|=8$, $|F|=5$, $|E|=?$
- For any planar graph $|V|+|F|=|E|+2$
- $8+5=|E|+2$
- $|E|=13-2=11$
- Ans : Number of edges is 11

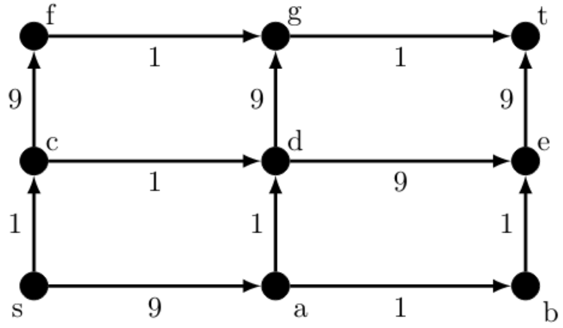
- ***GATE CS 2021 Set 2 | Question: 46**
- Consider the following directed graph:



- Which of the following is/are correct about the graph?
- A. The graph does not have a topological order
- B. A depth-first traversal starting at vertex *S* classifies three directed edges as back edges
- C. The graph does not have a strongly connected component
- D. For each pair of vertices *u* and *v*, there is a directed path from *u* to *v*
- **A)** The graph does not have a topological order , because there's a cycle in the bottom left corner of the graph
- **B)** Yes, there are only 3 back edges, if started from S.
- **C)** The graph does have a strongly connected component , it has cycle .
- **D)** False , you can observe and find that not all rectangular/square components forms a cycle.
- Ans : **(A) , (B)**

*GATE CS 2021 Set 2 | Question: 55

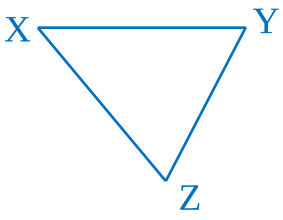
- In a directed acyclic graph with a source vertex s, the quality-score of a directed path is defined to be the product of the weights of the edges on the path. Further, for a vertex *v* other than s, the quality-score of *v* is defined to be the maximum among the quality-scores of all the paths from s to *v*. The quality-score of s is assumed to be 1.
- The sum of the quality-scores of all vertices on the graph shown above is ________



- Let Q(v) = quality-score of vertex v .
- Q(s)=1, Q(c)=1*1=1(s→c)
- Q(f)=1*9=9(s → c → f) Q(a)=1*9=9(s→a)
- Q(d)=9*1=9(s →a →d) Q(g)=9*9=81(s →d →g)
- Q(b)=9*1=9(s →a →b) Q(e)=9*9=81(s →d →e)
- Q(t)=81*9=729(s →e →t)
- Sum of the quality-scores of all vertices on the graph =
- 1+1+9*4+81*2+729=929
- Ans : 929

- ***GATE CS 2022 | Question: 20**
- Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is ________________ .
- If the graph is connected, then maximum edges=10*(10-1)/2=45
- But the graph is disconnected .
- Let's make 2 component ,one with 9 vertices and other one vertex .
- So maximum edges=9*8/2=36
- Ans :36

- ***GATE CS 2022 | Question: 27**
- Consider a simple undirected unweighted graph with at least three vertices. If A is the adjacency matrix of the graph, then the number of 3–cycles in the graph is given by the trace of A. A^3 B. A^3 divided by 2 C. A^3 divided by 3 D. A^3 divided by 6



A	X	Y	Z
X	0	1	1
Y	1	0	1
Z	1	1	0

*

A	X	Y	Z
X	0	1	1
Y	1	0	1
Z	1	1	0

=

A^2	X	Y	Z
X	2	1	1
Y	1	2	1
Z	1	1	2

*

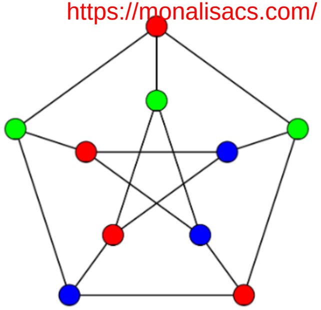
A	X	Y	Z
X	0	1	1
Y	1	0	1
Z	1	1	0

=

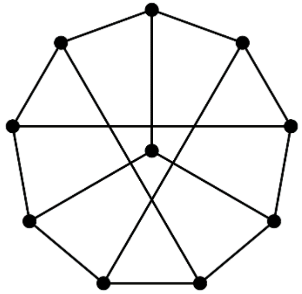
A^3	X	Y	Z
X	2	3	3
Y	3	2	3
Z	3	3	2

- Trace of A^3=6
- Number of 3-cycles in graph =1=A^3 divided by 6
- Ans : D. A^3 divided by 6

- ***GATE CS 2022 | Question: 40**
- The following simple undirected graph is referred to as the Peterson graph.



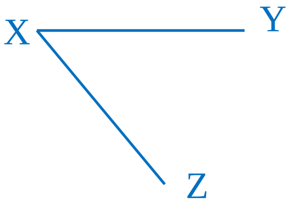
- Which of the following statements is/are TRUE?
- A. The chromatic number of the graph is 3.
- B. The graph has a Hamiltonian path.
- C. The following graph is isomorphic to the Peterson graph.



- D. The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)
- (A) 3 colors are sufficient and necessary to color it. **True**
- (B) Hamiltonian path exists but not Hamiltonian cycle. **True**
- (C) Graph present in Option C is isomorphic to Peterson graph .**True**
- (D)It has largest independent vertex size = 4 (red colored vertices in graph) . **False**
- Ans : A,B,C

- ## *GATE CS 2022 | Question: 42

- Which of the properties hold for the adjacency matrix A of a simple undirected unweighted graph having n vertices?



- A. The diagonal entries of A^2 are the degrees of the vertices of the graph.
- B. If the graph is connected, then none of the entries of $A^{n-1}+I_n$ can be zero.
- C. If the sum of all the elements of A is at most $2(n-1)$, then the graph must be acyclic.
- D. If there is at least a 1 in each of A's rows and columns, then the graph must be connected.
- (A) True
- (B) A^2+I_3
- False
- (C) sum=6
- But It have cycle
- False
- (D) False
- Ans : (A)

A^2	X	Y	Z
X	2	0	0
Y	0	1	1
Z	0	1	1

+

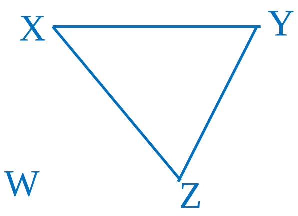
I_3	X	Y	Z
X	1	0	0
Y	0	1	0
Z	0	0	1

=

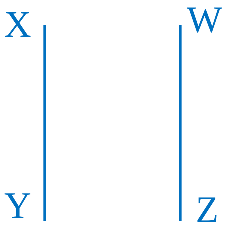
	X	Y	Z
X	3	0	0
Y	0	2	1
Z	0	1	2

A^2	X	Y	Z
X	2	0	0
Y	0	1	1
Z	0	1	1

A	X	Y	Z
X	0	1	1
Y	1	0	0
Z	1	0	0



A	X	Y	Z	W
X	0	1	1	0
Y	1	0	1	0
Z	1	1	0	0
W	0	0	0	0



A	X	Y	Z	W
X	0	1	0	0
Y	1	0	0	0
Z	0	0	0	1
W	0	0	1	0

- ***GATE CS 2023 | Question: 45**
- Let G be a simple, finite, undirected graph with vertex set $\{v_1, \ldots, v_n\}$. Let $\Delta(G)$ denote the maximum degree of G and let $N=\{1,2,\ldots\}$ denote the set of all possible colors. Color the vertices of G using the following greedy strategy: for $i=1,\ldots,n$
- $\text{color}(v_i) \leftarrow \min\{j \in N : \text{no neighbour of } v_i \text{ is colored } j\}$
- Which of the following statements is/are TRUE?
- A. This procedure results in a proper vertex coloring of G.
- B. The number of colors used is at most $\Delta(G)+1$.
- C. The number of colors used is at most $\Delta(G)$.
- D. The number of colors used is equal to the chromatic number of G.
- (A) True , The procedure results in a proper vertex coloring of G.
- (B) True , In complete graph color used are n and degree=n-1.
- (C) False , The number of colors used can be at most $\Delta(G) + 1$.
- (D) False , The number of colors used is not guaranteed to be equal to the chromatic number of G
- Ans : (A) ,(B)

- ***GATE CS 2024 | Set 1 | Question: 41**
- The chromatic number of a graph is the minimum number of colours used in a proper colouring of the graph. Let G be any graph with n vertices and chromatic number k. Which of the following statements is/are always TRUE?
- (A) G contains a complete subgraph with k vertices
- (B) G contains an independent set of size at least n/k
- (C) G contains at least $k(k-1)/2$ edges
- (D) G contains a vertex of degree at least k
- (A) False ,because chromatic number 'k' not necessarily means that graph contains a clique on 'k' vertices.
- Chromatic Number of a Cycle Graph on odd vertices is 3 but it doesn't contain any complete subgraph on 3 vertices.
- (B) True , If there are 'k' color classes and n vertices, then one color class size is atleast n/k.
- (C) True , Min number of edges in a graph having k Chromatic Number is { k(k-1) / 2 }.
- (D) False , because take a cycle graph on odd vertices, chromatic number is 3 but degree is not 3 for any vertex.
- In complete graph, degree for every vertex is n-1 but chromatic number is n
- Ans : (B),(C)

- ***GATE CS 2024 | Set 1 | Question: 50**
- The number of edges present in the forest generated by the DFS traversal of an undirected graph G with 100 vertices is 40. The number of connected components in G is ________.
- Forest have no crossing edges hence it is a planar graph .
- |V|=100 , |E|=40 ,R=1 unbounded region ,Component (K)=?
- |V|+|R|=|E|+K+1
- 100+1=40+K+1
- K=60
- Ans : 60

- ***GATE CS 2024 | Set 2 | Question: 7**
- Let A be the adjacency matrix of a simple undirected graph G. Suppose A is its own inverse. Which one of the following statements is *always* TRUE?
- (A)G is a cycle
- (B)G is a perfect matching
- (C)G is a complete graph
- (D)There is no such graph
- $A=A^{-1}$
- (A) G is not a cycle
- (B) If every vertex belongs to exactly one of the matching edges, we say the matching is perfect .
- G is a perfect Matching
- (C)G is not a complete graph
- (D)There are such graph
- Ans: (B) G is a perfect matching

- ***GATE CS 2024 | Set 2 | Question: 50**
- The chromatic number of a graph is the minimum number of colours used in a proper colouring of the graph. The chromatic number of the following graph is __________.
- The chromatic number of the given graph is 2
- **Ans : 2**