Algorithms

Chapter 6: Greedy Method

GATE CS PYQ

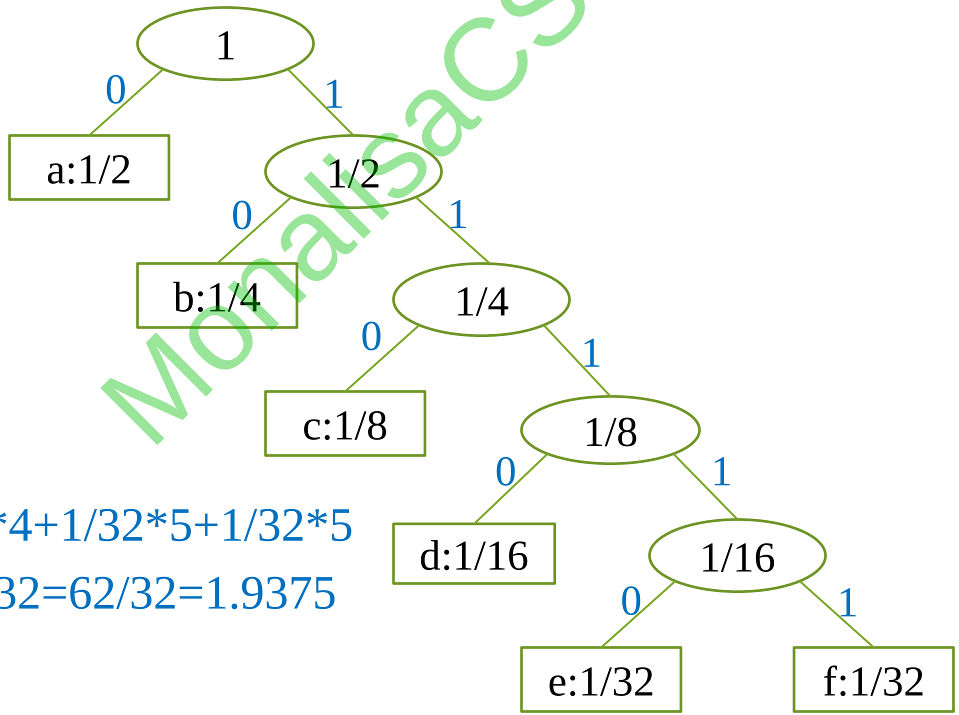
Solved by Monalisa

Section 5: Algorithms

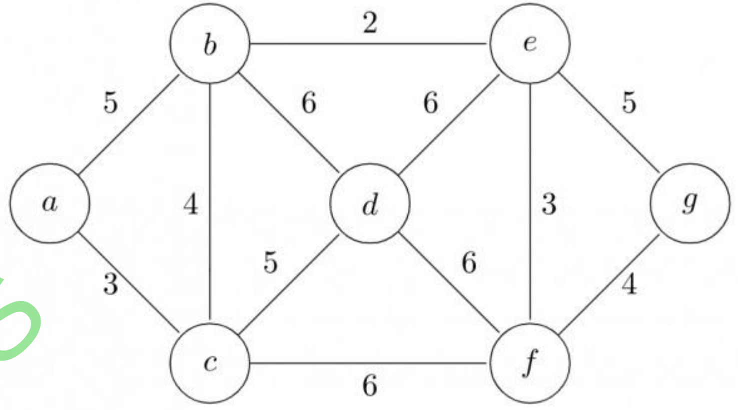
- Searching, sorting, hashing. Asymptotic worst case time and space complexity. Algorithm design techniques : greedy, dynamic programming and divide-and-conquer . Graph traversals, minimum spanning trees, shortest paths
- Chapter 1:Algorithim Analysis:-Algorithm intro , Order of growth ,Asymptotic notation, Time complexity, space complexity, Analysis of Recursive & non recursive program, Master theorem]
- Chapter 2:Brute Force:-Sequential search, Selection Sort and Bubble Sort , Radix sort, Depth first Search and Breadth First Search.
- Chapter 3: Decrease and Conquer :- Insertion Sort, Topological sort,Binary Search .
- Chapter 4: Divide and conquer:-Min max problem , matrix multiplication ,Merge sort ,Quick Sort , Binary Tree Traversals and Related Properties .
- Chapter 5: Transform and conquer:- Heaps and Heap sort, Balanced Search Trees.
- Chapter 6: Greedy Method:-knapsack problem , Job Assignment problem, Optimal merge, Hoffman Coding, minimum spanning trees, Dijkstra's Algorithm.
- Chapter 7: Dynamic Programming:-The Bellman-Ford algorithm ,Warshall's and Floyd's Algorithm ,Rod cutting, Matrix-chain multiplication ,Longest common subsequence ,Optimal binary search trees
- Chapter 8: Hashing.
- Reference : Introduction to Algorithms by Thomas H. Cormen
- Introduction to the Design and Analysis of Algorithms, by Anany Levitin
- My Note

- Chapter 6: Greedy Method:-
- knapsack problem
- **2018 | Q: 48**
- Job Sequencing with Deadlines,
- **2021 Set 1 | Q: 40**
- Optimal merge,
- **2014 Set 2 | Q: 38**
- Hoffman Coding,
- **2021 Set 2 | Q: 26, 2017 Set 2 | Q: 50**
- Minimum spanning trees,
- **2022 | 39, 2022 | 48,**
- Dijkstra's Algorithm.

- **GATE CS 2007 | Question: 77**
- Suppose the letters a,b,c,d,e,f have probabilities 1/ 2 ,1/4 ,1/8 ,1/16,1/32,1/32, respectively.
- What is the average length of the Huffman code for the letters a,b,c,d,e,f?
- (A)3 (B)2.1875 (C)2.25 (D)1.9375
- Prefix code Code length
- a=0 1
- b=10 2
- c=110 3
- d=1110 4
- e=11110 5
- f=11111 5
- Avg length of code
- =1/2*1+1/4*2+1/8*3+1/16*4+1/32*5+1/32*5
- =1/2+1/2+3/8+1/4+5/32+5/32=62/32=1.9375
- Ans : (D)



- **GATE CS 2009 | Question: 38**
- Consider the following graph:



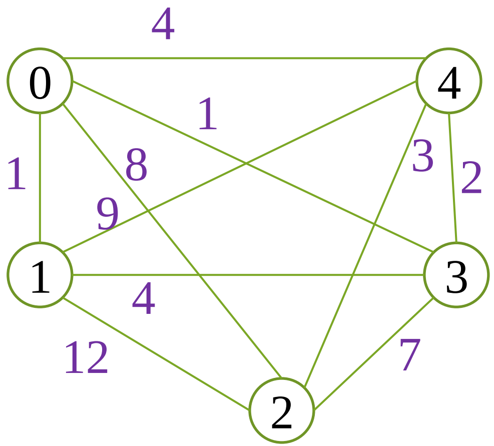
- Which one of the following is NOT the sequence of edges added to the minimum spanning tree using Kruskal's algorithm?
- A.(b, e) (e, f) (a, c) (b, c) (f, g) (c, d)
- B.(b, e) (e, f) (a, c) (f, g) (b, c) (c, d)
- C.(b, e) (a, c) (e, f) (b, c) (f, g) (c, d)
- D.(b, e) (e, f) (b, c) (a, c) (f, g) (c, d)
- Edges in sorted order={2,3,3,4,4,5,5,5,6,6,6,6 }
- 1st (b,e) weight 2
- 2nd and 3rd (a,c) or (e,f) weight 3
- In option D 3rd edge is (b,c) weight 4.
- In Kruskal's algorithm, edges should be added in non-decreasing order of weight.
- Ans: D.(b, e) (e, f) (b, c) (a, c) (f, g) (c, d)

GATE CS 2010 | Question: 50

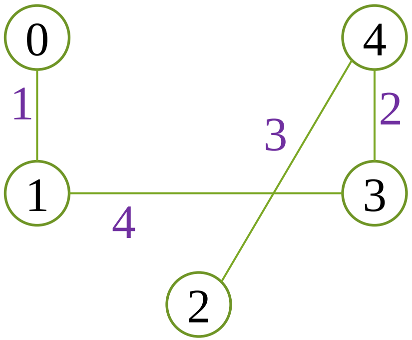
- Consider a complete undirected graph with vertex set {0,1,2,3,4}. Entry W_{ij} in the matrix W below is the weight of the edge $\{i,j\}$

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

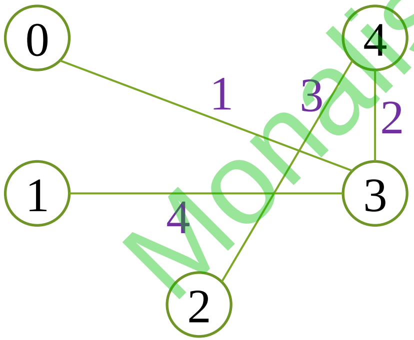
- What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?
- A)7 B)8 C)9 D)10



- Prim's MST



- Kruskal's MST



- 1+2+3+4=10
- Ans:D)10
- If vertex 0 is not leaf node then sum 1+1+2+3 = 7

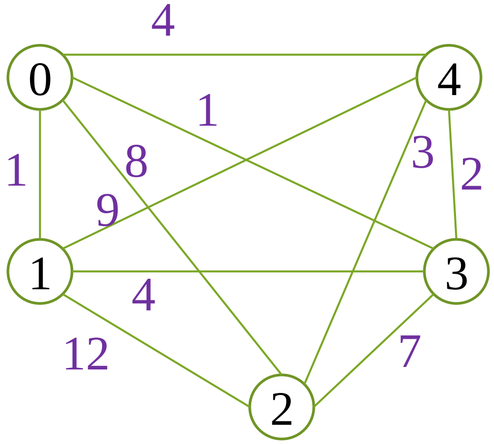
GATE CS 2010 | Question: 51

- Consider a complete undirected graph with vertex set {0,1,2,3,4}. Entry W_{ij} in the matrix W below is the weight of the edge $\{i,j\}$
- What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

- (A)7 (B)8 (C)9 (D)10

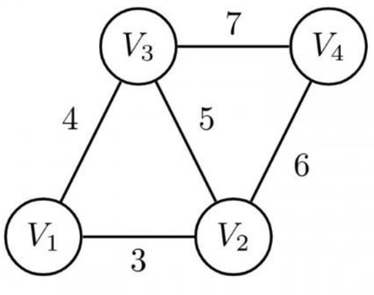
Edges	Weight
(1-2)	12
(1-0-2)	1+8=9
(1-0-4-2)	1+4+3=8
(1-0-3-2)	1+1+7=9



- If we apply Dijkstra's Alg then 1-0-3-4-2=7 but 4 edges so ans will be 8
- Ans : (B) 8

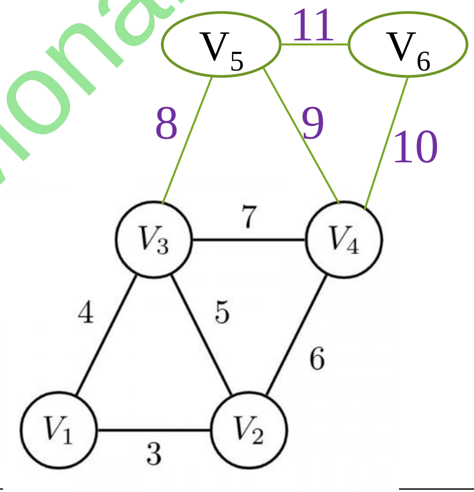
GATE CS 2011 | Question: 54

- An undirected graph G (V,E) contains n (n>2) nodes named $v_1, v_2, \ldots, v_n$. Two nodes v_i, v_j are connected if and only if $0 < |i-j| \leq 2$. Each edge (v_i, v_j) is assigned a weight $i+j$. A sample graph with n=4 is shown below.



- What will be the cost of the minimum spanning tree (MST) with n nodes?
- A. $\frac{1}{12}(11n^2-5n)$ B. n^2-n+1 C. $6n-11$ D. $2n+1$
- For n=3 cost of MST 3+4=7
- For n=4 cost of MST 3+4+6=13
- For n=5 cost of MST 3+4+6+8=21
- 1+2+2(2+3…+(n-1))
- $1+2(1+2+\ldots(n-1))=1+2\frac{(n-1)(n)}{2}=1+n^2-n= n^2-n+1$
- Ans : B. n^2-n+1

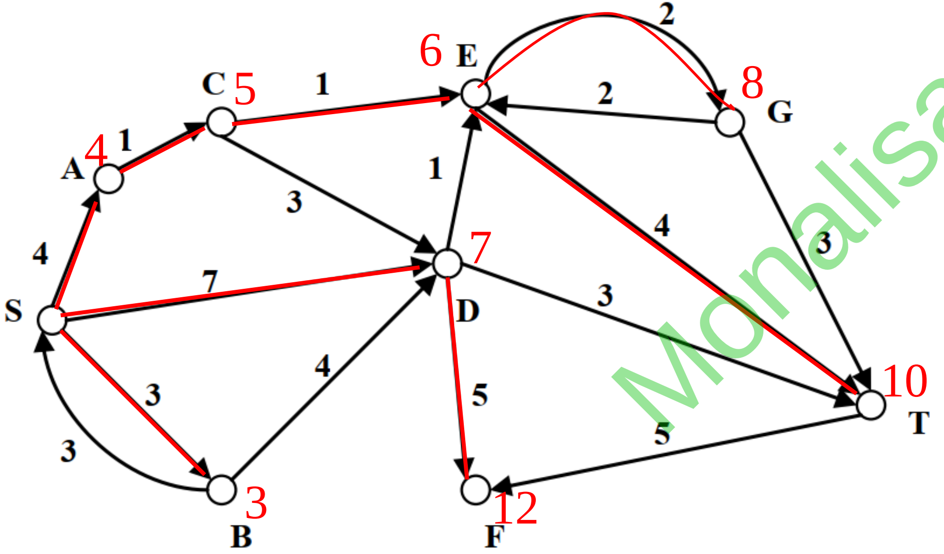
- **GATE CS 2011 | Question: 55**
- An undirected graph G(V,E) contains n(n>2) nodes named $v_1, v_2, \ldots, v_n$. Two nodes v_i, v_j are connected if and only if $0 < |i-j| \leq 2$. Each edge (v_i, v_j) is assigned a weight $i+j$. A sample graph with n=4 is shown below.
- The length of the path from v_5 to v_6 in the MST of previous question with n=10 is
- A.11 B.25 C.31 D.41
- Length of the path from v_5 to v_6 in the MST
- 8+4+3+6+10=31
- Ans: C.31



- **GATE CS 2012 | Question: 29**
- Let G be a weighted graph with edge weights greater than one and G′ be the graph constructed by squaring the weights of edges in G. Let T and T′ be the minimum spanning trees of G and G′, respectively, with total weights t and t′. Which of the following statements is **TRUE**?
- A) $T' = T$ with total weight $t' = t^2$
- B) $T' = T$ with total weight $t' < t^2$
- C) $T' \neq T$ but total weight $t' = t^2$
- D) None of the above
- When the edge weights are squared the minimum spanning tree structure won't change.
- $t' < t^2$, because sum of squares is always less than the square of the sums except for a single element case.
- Hence, B is the general answer and A is also true for a single edge graph. Hence, in GATE 2012, marks were given to all.
- Ans :B) $T' = T$ with total weight $t' < t^2$

GATE CS 2012 | Question: 40

- Consider the directed graph shown in the figure below. There are multiple shortest paths between vertices S and T. Which one will be reported by Dijkstra's shortest path algorithm? Assume that, in any iteration, the shortest path to a vertex *v* is updated only when a strictly shorter path to *v* is discovered.
- (A) SDT (B) SBDT (C) SACDT (D) SACET

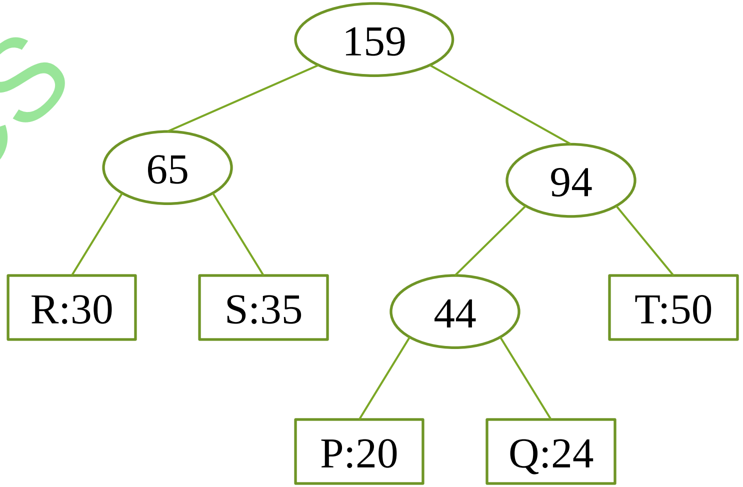


- DIJKSTRA(*G*, *w*, *s*)

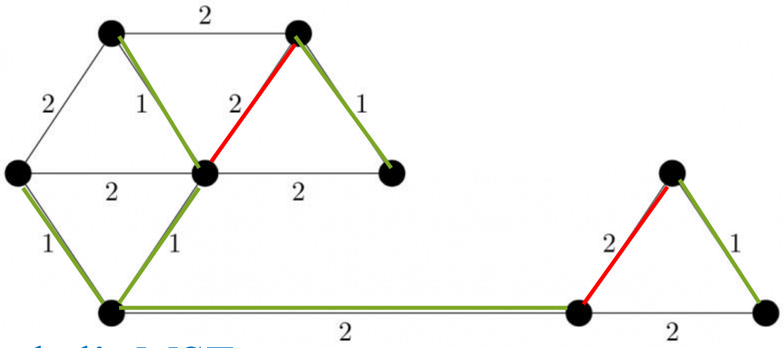
```
INITIALIZE-SINGLE-SOURCE (G, s)
S=Ø
Q= G.V
while Q≠ Ø
    u= EXTRACT-MIN(Q)
    S=S ∪ {u}
    for each vertex v ∈G . Adj[u]
        RELAX{u ,v ,w }
```

- Ans : (D) SACET

- **GATE CS 2014 Set 2 | Question: 38**
- Suppose P,Q,R,S,T are sorted sequences having lengths 20,24,30,35,50 respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is ____.
- Number of comparisons =m+n-1
- Total comparisons=43+64+93+158=358
- Ans : 358

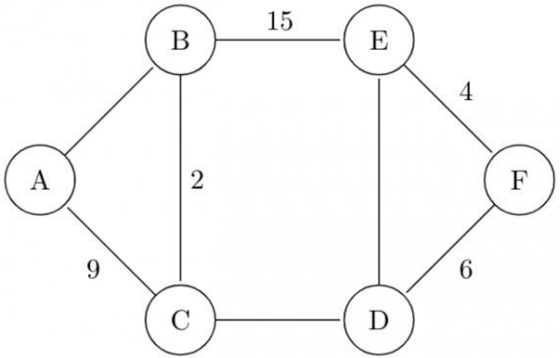


- **GATE CS 2014 Set 2 | Question: 52**
- The number of distinct minimum spanning trees for the weighted graph below is ______



- Kruskal’s MST
- Chose any one 2 from these three 2. 3 options
- Chose any one 2 from these two 2. 2 options
- Total =3*2=6 possibilities
- 6 distinct MST
- Ans : 6

- **GATE CS 2015 Set 1 | Question: 43**
- The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (**MST**) is of weight 36 and contains the edges: {(A,C),(B,C),(B,E),(E,F),(D,F)}. The edge weights of only those edges which are in the **MST** are given in the figure shown below. The minimum possible sum of weights of all 8 edges of this graph is________.



- In ABC. AC and BC are part of MST.
- AB is not included means AB should be greater than max(AC,BC)
- So, AB>9.
- Let Min AB=10
- Similarly, in DEF,ED>6.Let min DE =7
- Now for cycle BCDE,CD>15.Let min CD=16
- So, minimum possible sum of AB,CD,DE will be 10+7+16=33.
- Total sum of edge weights =33+36=69
- Ans : 69

- **GATE CS 2015 Set 3 | Question: 40**
- Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes ______.
- |V| = 100
- |E| = 99
- If each edges increased by five then weight of MST increase by 99*5=495
- New weight of MST=500+495=995
- Ans : 995

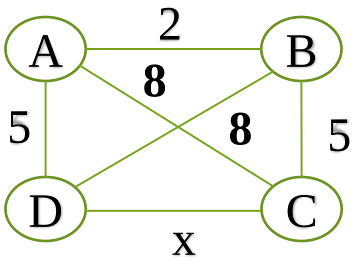
GATE CS 2016 Set 1 | Question: 14

- Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are TRUE?
- P: Minimum spanning tree of G does not change.
- Q: Shortest path between any pair of vertices does not change.
- A) P only B)Q only C)Neither P nor Q D)Both P and Q
- P: MST does not change,True
- Q: The shortest path may change. False
- There may be different number of edges in different paths from s to t.
- Let shortest path be of weight 20 and has 2 edges.
- Let another path with weight 25 and has 1 edge.
- If every edge weight is increased by the 10
- Then 20+2*10=40 shortest path increase
- Other path 25+10=35 so shortest path will change.
- Ans : A) P only

- **GATE CS 2016,Set-1, Question: 38**
- Consider the weighted undirected graph with 4 vertices, where the weight of edge {i, j} is given by the entry W_{ij} in the matrix W

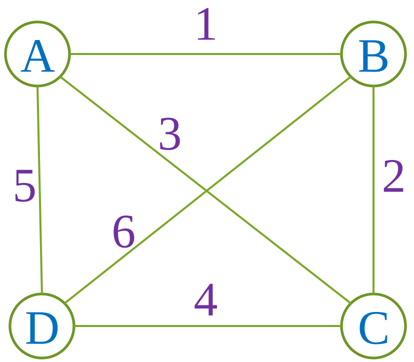
$$W=\begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

- The largest possible integer value of x, for which at least one shortest path between some pair of vertices will contain the edge with weight x is ________.

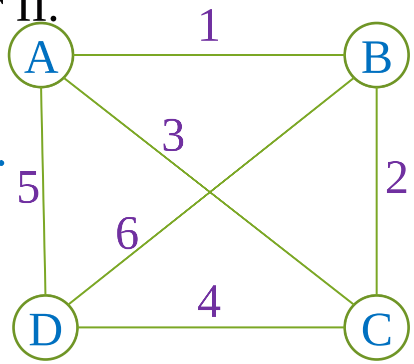


- The shortest path from C to D is of weight 12 (C-B-A-D)
- So largest possible value for x is 12
- Ans :12

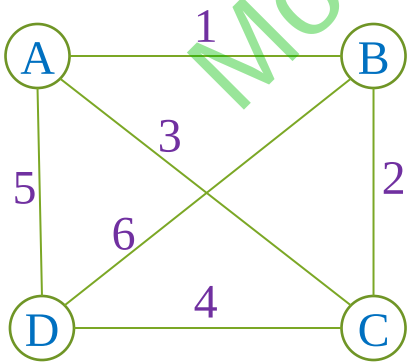
- **GATE CS 2016 Set 1 | Question: 39**
- Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1,2,3, 4,5, and 6. The maximum possible weight that a minimum weight spanning tree of G can have is __________ .
- |V|= 4,
- So in MST |E|=3
- If we use Kruskal's Algorithm
- 1,2 we will select , may skip 3 due to cycle and select 4.
- So maximum possible weight of MST =1+2+4=7
- Ans : 7



- **GATE CS 2016 Set 1 | Question: 40**
- G=(V,E) is an undirected simple graph in which each edge has a distinct weight, and e is a particular edge of G. Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?
- I.If e is the lightest edge of some cycle in G, then every MST of G includes e.
- II.If e is the heaviest edge of some cycle in G, then every MST of G excludes e.
- (A) I only. (B)II only. (C)Both I and II. (D)Neither I nor II.
- **I.** False, as we may need to skip e due to creating cycle.
- In graph AC is the lightest edge of ACD cycle but we are not including it.
- II. True, If heaviest edge is in cycle then we will always exclude that,
- We can have other choice of low cost edges.
- (If edge weights are not distinct even the heaviest edge can be part of the MST)
- Ans :B



- **GATE CS 2017 Set 1 | Question: 26**
- Let G=(V,E) be any connected, undirected, edge-weighted graph. The weights of the edges in E are positive and distinct. Consider the following statements:
- I. Minimum Spanning Tree of G is always unique.
- II. Shortest path between any two vertices of G is always unique.
- Which of the above statements is/are necessarily true?
- (A) I only (B) II only (C)both I and II (D)neither I nor II
- I. True, as all weights of the edges are distinct.
- II. False, Shortest Path can be different even if the edges are distinct.
- Ans : (A) I only

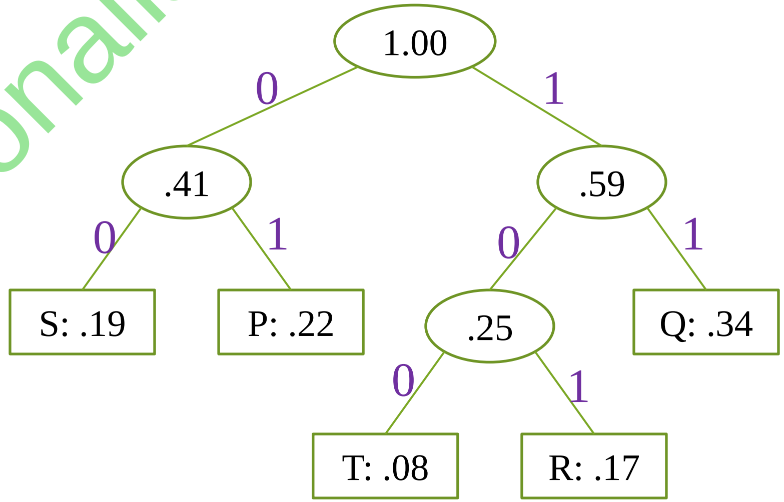


- **GATE CS 2017 Set 2 | Question: 50**
- A message is made up entirely of characters from the set X = {*P* ,*Q* ,*R* ,*S* ,*T*}. The table of probabilities for each of the characters is shown below:
- If a message of 100 characters over *X* is encoded using Huffman coding, then the expected length of the encoded message in bits is ______.

Character	Probability
P	.22
Q	.34
R	.17
S	.19
T	.08
Total	1.00

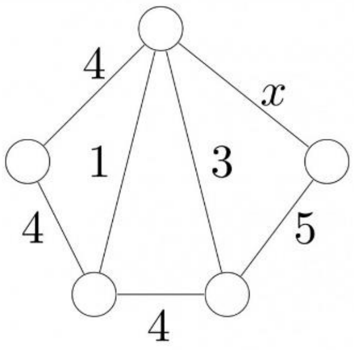
- Probabilities in ascending order : .08 , .17 , .19 , .22 , .34
- .19 , .22 ,.25, .34
- .25, .34,.41
- .41,.59

	Prefix code	Code length
P:	01	2
Q:	11	2
R:	101	3
S:	00	2
T:	100	3



- Expected length of the encoded char
- .22*2+.34*2+.17*3+.19*2+.08*3=2.25
- Expected length of encoded message =100*2.25=225
- Ans :225

- **GATE CS 2018 | Question: 47**
- Consider the following undirected graph G:



- Choose a value for x that will maximize the number of minimum weight spanning trees (MWSTs) of G. The number of MWSTs of G for this value of x is _____.
- Number of possible MSTs increase with non distinct edges.
- If x=5
- Number of MST =2×2=4 [2 for two 4s ,one 4 forming cycle so skip .2 for two 5s]
- If x=4 ,Number of MST =2
- If x=3 ,Number of MST=2
- If x=1, Number of MST =2
- Ans: 4

- **GATE CS 2018 | Question: 48**
- Consider the weights and values of items listed below. Note that there is only one unit of each item. The task is to pick a subset of these items such that their total weight is no more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt}. A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy}. The value of V_{opt} -V_{greedy} is____.

Item number	Weight (in Kgs)	Value(in rupees)
1	10	60
2	7	28
3	4	20
4	2	24

- $Greedy_{value}$: $x_1=1$, $x_2=0$, $x_3=0$, $x_4=0$
- $\sum_{i=1}^{n} w_i x_i$= 10*1+7*0 +4*0+2*0=10<11
- $\sum_{i=1}^{n} v_i x_i$= 60*1+28*0+20*0+24*0=60,
- V_{opt}= 60
- $Greedy_{value/weight}$: v_1/w_1 =60/10 =6 ,
- v_2/w_2 =28/7 =4, v_3/w_3 = 20/4=5 , v_4/w_4 = 24/2=12 , descending order 4, 1, 3 ,2
- $x_1=0, x_2=0, x_3=1, x_4=1$
- $\sum_{i=1}^{n} w_i x_i$= 10*0+7*0 +4*1+2*1=6<11
- $\sum_{i=1}^{n} v_i x_i$= 60*0+28*0+20*1+24*1=44, V_{greedy} =44
- $V_{opt} - V_{greedy}$ =60-44=16 , Ans : 16

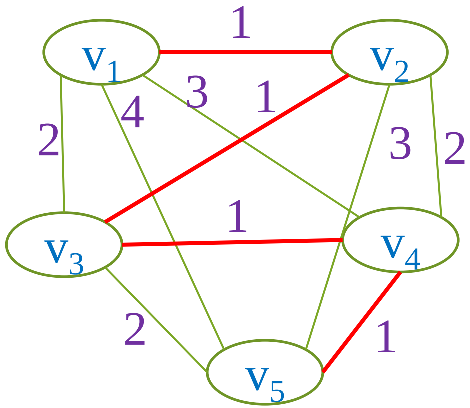
- **GATE CS 2019 | Question: 38**
- Let G be any connected, weighted, undirected graph.
- I. G has a unique minimum spanning tree, if no two edges of G have the same weight.
- II. G has a unique minimum spanning tree, if, for every cut of G, there is a unique minimum-weight edge crossing the cut.
- Which of the following statements is/are TRUE?
- A. I only B.II only C.Both I and II D.Neither I nor II
- I.True , If edge weights are distinct then there exist unique MST.
- II.True,
- Theorem : Let G=(V, E) be a connected, undirected graph with a real-valued weight function w defined on E. Let A be a subset of E that is included in some minimum spanning tree for G, let (S, V-S) be any cut of G that respects A, and let (u,v) be a light edge crossing (S,V-S). Then, edge (u,v) is safe for A.
- Ans : C.Both I and II

- **GATE CS 2020 | Question: 31**
- Let G=(V,E) be a weighted undirected graph and let T be a Minimum Spanning Tree (MST) of **G** maintained using adjacency lists. Suppose a new weighed edge $(u,v) \in V \times V$ is added to G. The worst case time complexity of determining if T is still an MST of the resultant graph is
- A.Θ(|E|+|V|) B.Θ(|E||V|) C.Θ(E|log|V|) D.Θ(|V|)
- We can do this in O(|V|) in the following way:

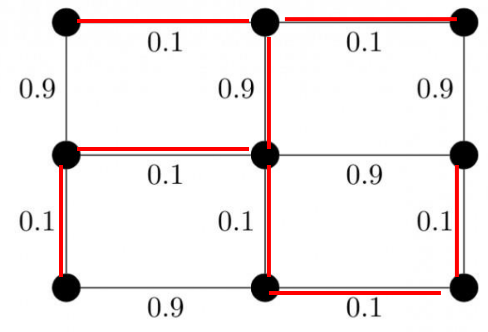
1. Run BFS in T from u to v to detect the edge with maximum value in that path. −O(|V|).
2. If the weight of that edge is greater than the weight of the edge you're trying to add, remove that old edge and add the new one.
3. Otherwise, do nothing, because the new edge would not improve the MST. −O(1).

- Ans : D.Θ(|V|)

- **GATE CS 2020 | Question: 49**
- Consider a graph $G=(V,E)$, where $V=\{v_1,v_2,\ldots,v_{100}\}$, $E=\{(v_i,v_j) \mid 1\le i<j\le 100\}$, and weight of the edge (v_i,v_j) is $|i-j|$. The weight of minimum spanning tree of G is _____ .
- Cost of MST=1+1+1+1=4
- |V|=5 ,|E|=|V|-1=4,
- Cost of MST=|V|-1=4
- |V|=100,|E|=100-1=99
- Weight of spanning tree 99∗1=99
- Ans : 99



- **GATE CS 2021 Set 1 | Question: 17**
- Consider the following undirected graph with edge weights as shown:



- The number of minimum-weight spanning trees of the graph is____.
- Lets construct MST by using Kruskal’s Algorithm
- We got 2 subtrees, to make it connected we need to select a edge .9 from these 3 options.
- Three .9 are present which are not making cycle.
- So 3 MST possible .
- Ans :3

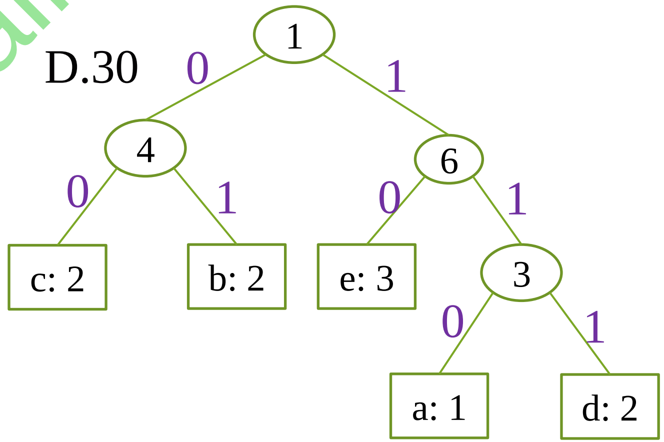
- **GATE CS 2021 Set 2 | Question: 1**
- Let G be a connected undirected weighted graph. Consider the following two statements.
- S_1: There exists a minimum weight edge in G which is present in every minimum spanning tree of G.
- S_2: If every edge in G has distinct weight, then G has a unique minimum spanning tree.
- Which one of the following options is correct?
- A.Both S_1 and S_2 are true B.S_1 is true and S_2 is false
- C.S_1 is false and S_2 is true D.Both S_1 and S_2 are false
- S_1false, If we have multiple copies of minimum weight edge, then a specific small weighted edge is not guaranteed to be selected .
- S_2 true.
- Ans : C.S_1 is false and S_2 is true

- **GATE CS 2021 Set 2 | Question: 26**
- Consider the string **abbccddeee**. Each letter in the string must be assigned a binary code satisfying the following properties:

1. For any two letters, the code assigned to one letter must not be a prefix of the code assigned to the other letter.
2. For any two letters of the same frequency, the letter which occurs earlier in the dictionary order is assigned a code whose length is at most the length of the code assigned to the other letter.

- Among the set of all binary code assignments which satisfy the above two properties, what is the minimum length of the encoded string?
- A. 21 B.23 C.25 D.30
- Ans: B.23

Char, Frequency		Prefix code, length	
a	1	110	3
b	2	01	2
c	2	00	2
d	2	111	3
e	3	10	2

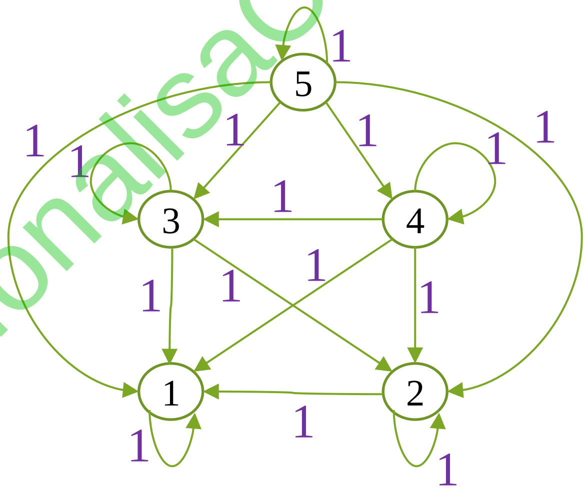


- Length of the encoded string=1*3+2*2+2*2+2*3+3*2=23

- **GATE CS 2022 | Question: 39**
- Consider a simple undirected weighted graph G, all of whose edge weights are distinct. Which of the following statements about the minimum spanning trees of G is/are TRUE?
- A. The edge with the second smallest weight is always part of any minimum spanning tree of G.
- B. One or both of the edges with the third smallest and the fourth smallest weights are part of any minimum spanning tree of G.
- C. Suppose $S \subseteq V$ be such that $S \neq \phi$ and $S \neq V$. Consider the edge with the minimum weight such that one of its vertices is in S and the other in $V \setminus S$. Such an edge will always be part of any minimum spanning tree of G.
- D.G can have multiple minimum spanning trees.
- **Option A :**True, Minimum Edge, Second Minimum Edge, both will be added in the MST by Kruskal Algorithm (Two edges cannot bring cycle)
- **Option B :**True, Minimum Edge, Second Minimum Edge, both will be added in the MST by Kruskal Algorithm , But third minimum edge might introduce cycle in the MST, and if so happens, we can skip third minimum edge in the MST and add fourth minimum edge in the MST.
- **Option C :**True ,a ***light edge*** is a safe edge and always part of minimum spanning tree of G
- **Option D :**False (Because all edge weights are distinct)
- Ans : A,B,C

GATE CS 2022 | Question: 48

- Let G(V,E) be a directed graph, where V={1,2,3,4,5} is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix A.
- A[*i*][*j*]={1,1≤j≤i≤5 0,otherwise
- A[*i*][*j*]=1 indicates a directed edge from node i to node j. A *directed spanning tree* of G, rooted at r ∈ V, is defined as a subgraph T of G such that the undirected version of T is a tree, and T contains a directed path from r to every other vertex in V. The number of such directed spanning trees rooted at vertex 5 is ______.
- |V|=5 vertices so in MST |E|=5-1= 4
- 5 is the root node
- With 5, 4nodes are adjacent
- So total 4!=24 MST possible.
- or,
- For 2 nodes : 1 MST
- For 3 nodes : 2*1=2 MST
- For 4 nodes : 3*2*1=6 MST
- For 5 nodes : 4*3*2*1=24 MST
- Ans : 24

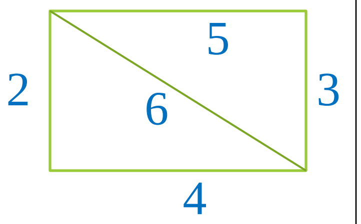


V	1	2	3	4	5
1	1	0	0	0	0
2	1	1	0	0	0
3	1	1	1	0	0
4	1	1	1	1	0
5	1	1	1	1	1

- ***GATE CS 2024 | Set 1 | Question: 24**
- The number of spanning trees in a *complete* graph of 4 vertices labelled A, B, C, and D is ___________.
- Number of spanning trees in a complete graph=n^{n-2}
- $4^2=16$
- Ans:16

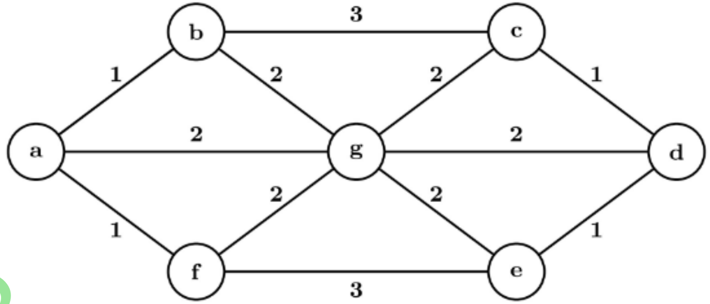
*GATE CS 2024 | Set 2 | Question: 41

- Let G be an undirected connected graph in which every edge has a positive integer weight. Suppose that every spanning tree in G has *even* weight. Which of the following statements is/are TRUE for *every* such graph G
- (A) All edges in G have even weight
- (B) All edges in G have even weight **OR** all edges in G have odd weight
- (C) In each cycle C in G, all edges in C have even weight
- (D) In each cycle C in G, either all edges in C have even weight **OR** all edges in C have odd weight



- Every spanning tree in G has *even* weight means sum of weight =even
- **(A)** wrong ,It can be odd too
- (B) wrong ,It can have mix of odd and even weight
- (C) wrong ,it can have even or odd weight cycle
- (D) Correct ,odd sum can be even too .
- Ans: (D) In each cycle C in G, either all edges in C have even weight **OR** all edges in C have odd weight.

- **GATE CS 2024 | Set 2 | Question: 49**
- The number of distinct minimum-weight spanning trees of the following graph is ______ .



- Let’s apply Kruskal’s Algorithm
- 1^{st} we will choose all 1’s
- Then We need to choose two 2’s in such a way that it will not create cycle and stay connected .
- One weight 2 edge from bg,ag,or fg $= {}^{3}C_{1} = 3$
- Other weight 2 edge from cg,dg,or eg $= {}^{3}C_{1} = 3$
- So total 3*3=9 minimum-weight spanning trees possible .
- Ans: 9