Discrete Mathematics
Chapter 2 : Set Theory

GATE CS PYQ
by Monalisa

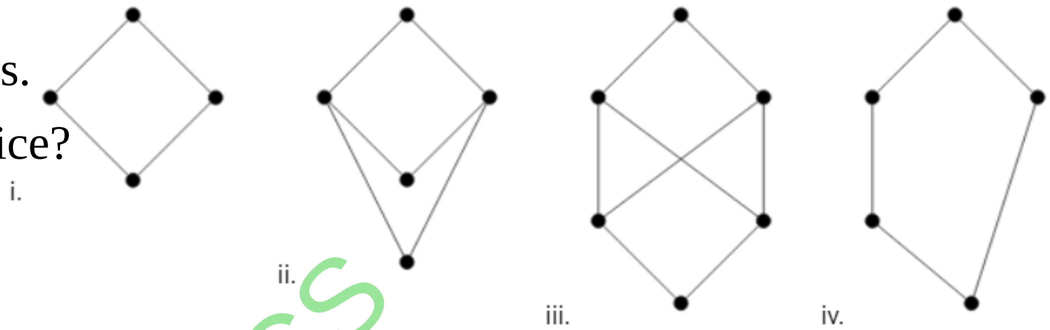
Section1: Engineering Mathematics

- **Discrete Mathematics**: Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations , generating functions.
- **Linear Algebra**: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.
- **Calculus**: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.
- **Probability and Statistics**: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

Chapter 2 : Set Theory

- 2.1 Sets (21,19,15
- 2.2 Set Operations(16,14,14,13
- 2.3 Functions (16,15,15
- 2.4 Sequences and Summations
- 2.5 Cardinality of Sets(18,15,14
- 2.6 Relations and Their Properties (21,20
- 2.7 n-ary Relations and Their Applications
- 2.8 Representing Relations
- 2.9 Closures of Relations (10,16
- 2.10 Equivalence Relations (19
- 2.11 Partial Orderings
- 2.12 Groups (23,

- ***GATE IT 2008 | Question: 28**
- Consider the following Hasse diagrams.



- Which all of the above represent a lattice?
- (A) (i) and (iv) only
- (B) (ii) and (iii) only
- (C) (iii) only (D) (i), (ii) and (iv) only
- A partially ordered set in which every pair of elements has both a least upper bound and a greatest lower bound is called a **lattice**
- i.Lattice
- ii. Not a lattice.
- iii. Not a lattice.
- iv. Lattice
- Ans: (A) (i) and (iv) only

- ***GATE CS 2009 | Question: 1**
- Which one of the following is **NOT** necessarily a property of a Group?
- A)Commutativity
- B)Associativity
- C)Existence of inverse for every element
- D)Existence of identity
- **Group:** closure, associative, identity, inverse.
- Ans : A)Commutativity

- ***GATE CS 2009 | Question: 4**
- Consider the binary relation R ={(x,y),(x,z),(z,x),(z,y)} on the set {x,y,z}. Which one of the following is **TRUE**?
- A. R is symmetric but NOT antisymmetric B. R is NOT symmetric but antisymmetric
- C. R is both symmetric and antisymmetric D. R is neither symmetric nor antisymmetric
- R ={(x,y),(x,z),(z,x),(z,y)}
- (y,x),(y,z) are not present hence not symmetric .
- (x,z),(z,x) both are present $x \neq z$ hence not antisymmetric
- Ans : D. R is neither symmetric nor antisymmetric

- *GATE CS 2009 | Question: 22
- For the composition table of a cyclic group shown below:

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

- Which one of the following choices is correct?
- A)a,b are generators B)b,c are generators
- C)c,d are generators D)d,a are generators
- a*a= a , a*a*a=a ,a is not a generator.
- b*b=a , b*b*b=b , b*b*b*b=a,b is not a generator .
- c*c=b , c*c*c=d , c*c*c*c=a , c is a generator .
- d*d=b , d*d*d=c , d*d*d*d=a , d is a generator .
- Ans : C)c,d are generators

- ***GATE CS 2010 | Question: 3**
- What is the possible number of reflexive relations on a set of 5 elements?
- (A)2^{10} (B)2^{15} (C)2^{20} (D)2^{25}
- Number of reflexive relations on a set of n elements= $2^{n^2 - n}$
- Number of reflexive relations on a set of 5 elements= $2^{5^2 - 5}$= $2^{25 - 5}$ =2^{20}
- Ans : (C)2^{20}

- ***GATE CS 2010 | Question: 4**
- Consider the set $S=\{1,w,w^2\}$, where w and w^2 are cube roots of unity. If $*$ denotes the multiplication operation, the structure $(S,*)$ forms a ______
- Closer
- Associative as multiplication is associative
- Identity element=1
- Inverse of $1=1, w= w^2, w^2 =w$
- It satisfy all properties of group .
- Ans : Group

	1	w	w^2
1	1	w	w^2
w	w	w^2	1
w^2	w^2	1	w

- ***GATE CS 2012 | Question: 37**
- How many onto (or surjective) functions are there from an *n*-element ($n \geq 2$) set to a 2-element set? A. 2^n B. $2^n - 1$ C. $2^n - 2$ D. $2(2^n - 2)$
- If |A|=m and |B|=n (m>n) then numbers of onto functions possible from A →B is
- $n^m - nc_1(n-1)^m + nc_2(n-2)^m - nc_3(n-3)^m + \ldots\ldots(-1)^n\, nc_{n-1}(1)^m$
- |A|=n ,|B|=2
- $2^n - 2c_1(2-1)^n$
- $2^n - 2$
- Ans : C. $2^n - 2$

- ***GATE CS 2013 | Question: 1**
- A binary operation ⊕ on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is **TRUE** about ⊕?
- (A)Commutative but not associative
- (B)Both commutative and associative
- (C)Associative but not commutative
- (D)Neither commutative nor associative
- $x \oplus y = x^2 + y^2$.
- Commutativity: $x \oplus y = y \oplus x$.
- $x \oplus y = x^2 + y^2 = y^2 + x^2 = y \oplus x$
  LHS = RHS. hence commutative.
- Associativity: $x \oplus (y \oplus z) = (x \oplus y) \oplus z$
- $x \oplus (y \oplus z) = x \oplus (y^2+z^2) = x^2+(y^2+z^2)^2$
- $(x \oplus y) \oplus z = (x^2+y^2) \oplus z = (x^2+y^2)^2+z^2$
- $x^2+(y^2+z^2)^2 \neq (x^2+y^2)^2+z^2$
- Hence not associative
- Ans : (A)Commutative but not associative

- ***GATE CS 2014 Set 1 | Question: 50**
- Let S denote the set of all functions f:$\{0,1\}^4 \rightarrow \{0,1\}$. Denote by N the number of functions from S to the set $\{0,1\}$. The value of $\log_2 \log_2 N$ is ________.
- $\{0,1\}^4$ contains 2^4 elements.
- $|S|=2^{2^4}$
- $|N|=2^{2^{2^4}}$
- $\log_2\log_2 N = \log_2\log_2(2^{2^{2^4}})$
- $=2^4=16$
- Ans : 16

- ***GATE CS 2014 Set 2 | Question: 50**
- Consider the following relation on subsets of the set *S* of integers between 1 and 2014. For two distinct subsets *U* and *V* of *S* we say *U*<*V* if the minimum element in the symmetric difference of the two sets is in U.
- Consider the following two statements:
- S_1: There is a subset of *S* that is larger than every other subset.
- S_2: There is a subset of *S* that is smaller than every other subset.
- Which one of the following is CORRECT?
- (A)Both S_1 and S_2 are true (B)S_1 is true and S_2 is false
- (C)S_2 is true and S_1 is false (D)Neither S_1 nor S_2 is true
- Symmetric difference of A and B (A−B)∪(B−A)=(A∪B)−(A∩B).
- U < V if the minimum element in the symmetric difference of the two sets is in U .
- Suppose S={1,2,3,4,5} Let U={1,2,3,4,5} and V={1,2,5} ,SD={3,4} U<V
- S is smaller than any other subset of S. **S_2 is true.**
- Now consider U=∅ and V={1,2} ,SD={1,2}
- The SD will always be equal to V. V<U when U is ∅.
- ∅ is greater than any other subset of S. **S_1 is also true.**
- Ans : (A)Both S_1 and S_2 are true

- ***GATE CS 2014 Set 3 | Question: 2**
- Let X and Y be finite sets and f:X→Y be a function. Which one of the following statements is TRUE?
- A. For any subsets A and B of X,$|f(A\cup B)|=|f(A)|+|f(B)|$
- B. For any subsets A and B of X,$f(A\cap B)=f(A)\cap f(B)$
- C. For any subsets A and B of X,$|f(A\cap B)|=\min\{|f(A)|,|f(B)|\}$
- D. For any subsets S and T of Y , $f^{-1}(S\cap T)=f^{-1}(S)\cap f^{-1}(T)$
- Let X={1,2,3} , Y={a ,b,c} ,f(1)=b,f(2)=a,f(3)=c,Let A={1,2},B={2,3}
- A. $|f(A\cup B)|=|f(1,2,3)|=3$, $|f(A)|+|f(B)|=2+2=4$,LHS≠RHS
- B. $f(A\cap B)=a$, $f(A)\cap f(B)=\{a,b\}\cap\{a,c\}=\{a\}$, LHS=RHS
- If we consider a function f(1)=a,f(2)=b,f(3)=a
- $f(A\cap B)=b$, $f(A)\cap f(B)=\{a,b\}\cap\{a,b\}=\{a,b\}$, LHS≠RHS
- C. $|f(A\cap B)|=1$,min {2,2}=2, LHS≠RHS
- D. Let S={a,b} T={b,c},$f^{-1}(S\cap T)=f^{-1}(b)=\{1\}$
- $f^{-1}(S)\cap f^{-1}(T)=\{1,2\}\cap\{1,3\}=\{1\}$
- Ans : D. For any subsets S and T of Y , $f^{-1}(S\cap T)=f^{-1}(S)\cap f^{-1}(T)$

- **GATE CS 2014 Set 3 | Question: 3**
- Let G be a group with 15 elements. Let L be a subgroup of G. It is known that L≠ G and that the size of L is at least 4. The size of L is ___________.
- O(G)=15
- O(L) can be 1,3,5,15
- But 4≤O(L)<15
- So O(L)=5
- Ans: 5

- ***GATE CS 2014 Set 3 | Question: 49**
- Consider the set of all functions $f:\{0,1,\ldots,2014\}\rightarrow\{0,1,\ldots,2014\}$ such that $f(f(i))=i$, for all $0\leq i\leq 2014$. Consider the following statements:
- P. For each such function it must be the case that for every $i, f(\mathrm{i})=i$.
- Q. For each such function it must be the case that for some $i, f(\mathrm{i})=i$.
- R. Each function must be onto.
- Which one of the following is CORRECT?
- (A)P,Q and R are true (B)Only Q and R are true
- (C)Only P and Q are true (D)Only R is true
- There can be two possibility
- (1) $f(\mathrm{i})=j,\ f(j)=i \Rightarrow f(f(i))=i$ for $i\neq \mathrm{j}$ (2) $f(\mathrm{i})=i$, $f(f(i))=i$
- $f(0)=1$, $f(1)=0$, $f(2)=2$, $f(3)=4$, $f(4)=3$, …. $f(2013)=2014$, $f(2014)=2013$
- So P false ,Q true.
- ‘i’ ranges from 0 to 2014, so, it takes 2015 possible values.
- domain and co – domain are exactly same.
- All co-domains are image of some domain.
- The function is onto and hence, R is definitely true.
- Ans : (B)Only Q and R are true

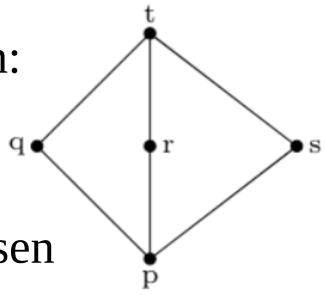
- **GATE CS 2014 Set 3 | Question: 50**
- There are two elements x,y in a group (G,∗) such that every element in the group can be written as a product of some number of x's and y's in some order. It is known that x∗x = y∗y = x∗y∗x∗y=y∗x∗y∗x=e where e is the identity element. The maximum number of elements in such a group is ____.
- x ,y ,xy ,yx are inverse of itself .
- x*y=x*e*y=x*(x*y*x*y)*y=(x*x)*y*x*(y*y)=y*x
- x*y=y*x
- G={(e,x,y,x*y),*}
- Ans :4

*	e	x	y	x*y
e	e	x	y	x*y
x	x	e	x*y	y
y	y	x*y	e	x
x*y	x*y	y	x	e

- ***GATE CS 2015 Set 1 | Question: 5**
- If g(x)=1-x and h(x)=$\frac{x}{x-1}$ then $\frac{g(h(x))}{h(g(x))}$ is :
- A. $\frac{h(x)}{g(x)}$ B. $\frac{-1}{x}$ C. $\frac{g(x)}{h(x)}$ D. $\frac{x}{(1-x)^2}$
- g(h(x))=g $(\frac{x}{x-1})$
- 1- $\frac{x}{x-1} = \frac{x-1-x}{x-1} = \frac{-1}{x-1}$
- $h(g(x))$ = h(1-x)= $\frac{1-x}{1-x-1} = \frac{1-x}{-x}$
- $\frac{g(h(x))}{h(g(x))} = \frac{-1}{x-1} / \frac{1-x}{-x} = \frac{x}{(x-1)(1-x)}$
- Ans A. $\frac{h(x)}{g(x)}$

- ***GATE CS 2015 Set 1 | Question: 16**
- For a set *A*, the power set of *A* is denoted by 2^A. If *A*={5,{6},{7}}, which of the following options are TRUE?
- I.$\emptyset \in 2^A$ II$\emptyset \subseteq 2^A$ III$\{5,\{6\}\} \in 2^A$ IV$\{5,\{6\}\} \subseteq 2^A$
- (A)I and III only (B)II and III only
- (C)I, II and III only (D)I, II and IV only
- P(A)={Ø,{5},{{6}},{{7}},{5,{6}},{5,{7}},{{6},{7}},{5,{6},{7}}}
- I. True , Ø belongs to power set .
- II.True , empty set is a subset of every set.
- III. True , {5,{6}} belongs to power set
- IV. False , {5,{6}} is a elements of power set not subset .
- {{5,{6}}} is subset of power set .
- Ans: (C)I, II and III only

- ***GATE CS 2015 Set 1 | Question: 34**
- Suppose L={p,q,r,s,t} is a lattice represented by the following Hasse diagram:



- For any x,y∈*L*, not necessarily distinct , x∨y and x∧y are join and meet of x,y, respectively. Let L^3={(x,y,z):x,y,z∈L} be the set of all ordered triplets of the elements of L. Let P_r be the probability that an element (x,y,z)∈L^3 chosen equiprobably satisfies x∨(y∧z)=(x∨y)∧(x∨z). Then
- (A) $P_r = 0$ (B) $P_r = 1$ (C) $0 < P_r \le \frac{1}{5}$ (D) $\frac{1}{5} < P_r < 1$
- $|L^3|$=Number of ways in which we can choose 3 elements from 5 with repetition =5*5*5=125.
- Now, when we take x = t, then the given condition for L is satisfied for any y and z.
- Here, y and z can be taken in 5 * 5 = 25 ways.
- Take x = r, y = p, z = p. x∨(y∧z)=r,(x∨y)∧(x∨z)=r
- Here also, the given condition is satisfied. So, $p_r > 25 / 125 > 1/5$.
- For x=q, y= r, z = s, q ∨ (r ∧ s) = q ∨ p = q, while (q ∨ r) ∧ (q ∨ s) = t ∧ t = t. So, $p_r \neq 1$.
- or, Its not a distributive lattice hence $p_r \neq 1$
- Ans : (D) $\frac{1}{5} < P_r < 1$

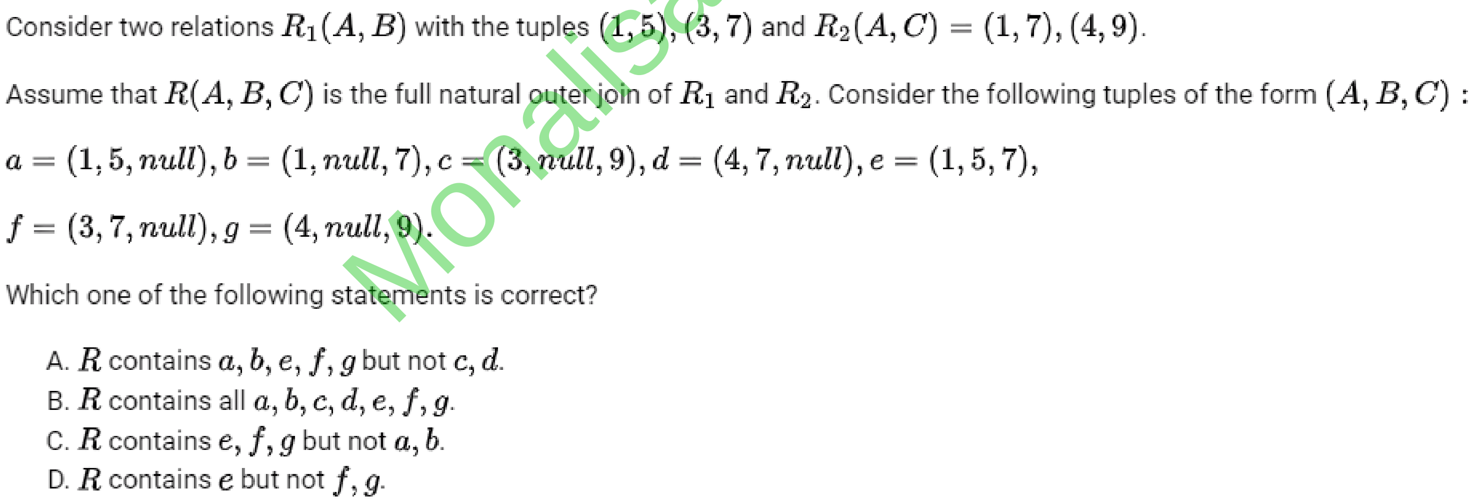
- ***GATE CS 2015 Set 2 | Question: 16**
- Let R be the relation on the set of positive integers such that *aRb* and only if *a* and *b* are distinct and let have a common divisor other than 1. Which one of the following statements about *R* is true?
- A. *R* is symmetric and reflexive but not transitive
- B. *R* is reflexive but not symmetric not transitive
- C. *R* is transitive but not reflexive and not symmetric
- D. *R* is symmetric but not reflexive and not transitive
- Not Reflexive : aRa ,a and a are not distinct.
- Symmetric : aRb and bRa possible
- Ex (6,4) (4,6) $4 \neq 6$,common divisor 2.
- Not Transitive :(3,6) (6,2) then (3,2)
- But 3 and 2 have no common divisor ..
- Ans: D. *R* is symmetric but not reflexive and not transitive

- ***GATE CS 2015 Set 2 | Question: 18**
- The cardinality of the power set of {0,1,2,…,10} is _______.
- Cardinality of set=11
- The cardinality of the power set $=2^{11}=2048$
- Ans :2048

GATE CS 2015 Set 2 | Question: 32

- Consider two relations R_1(A,B) with the tuples (1,5),(3,7) and R_2(A,C)=(1,7),(4,9).
- Assume that R(A,B,C) is the full natural outer join of R1 and □2. Consider the following tuples of the form (□,□,□):
- □=(1,5,□□□□),□=(1,□□□□,7),□=(3,□□□□,9),□=(4,7,□□□□),□=(1,5,7),
- □=(3,7,□□□□),□=(4,□□□□,9).
- Which one of t

1. □ contains □,□
2. □ contains all □
3. □ contains □,□
4. □ contains □ b


Consider two relations $R_1(A, B)$ with the tuples $(1, 5), (3, 7)$ and $R_2(A, C) = (1, 7), (4, 9)$.

Assume that $R(A, B, C)$ is the full natural outer join of R_1 and R_2. Consider the following tuples of the form (A, B, C) :

$a = (1, 5, null), b = (1, null, 7), c = (3, null, 9), d = (4, 7, null), e = (1, 5, 7),$

$f = (3, 7, null), g = (4, null, 9).$

Which one of the following statements is correct?

A. R contains a, b, e, f, g but not c, d.
B. R contains all a, b, c, d, e, f, g.
C. R contains e, f, g but not a, b.
D. R contains e but not f, g.

- ***GATE CS 2015 Set 2 | Question: 40**
- The number of onto functions (surjective functions) from set X={1,2,3,4} to set Y={a,b,c} is _______.
- If |X|=m and |Y|=n (m>n) then numbers of onto functions possible from X → Y is $n^m - nc_1(n-1)^m + nc_2(n-2)^m - nc_3(n-3)^m + \ldots\ldots(-1)^n nc_{n-1}(1)^m$
- m=4,n=3
- $n^m - nc_1(n-1)^m + nc_2(n-2)^m - nc_3(n-3)^m + \ldots\ldots(-1)^n nc_{n-1}(1)^m$
- $=3^4 - 3c_1(3-1)^4 + 3c_2(3-2)^4$
- =81-3*16+3
- =81-48+3
- =84-48
- =36
- Ans : 36

- ***GATE CS 2015 Set 2 | Question: 54**
- Let *X* and *Y* denote the sets containing 2 and 20 distinct objects respectively and *F* denote the set of all possible functions defined from *X* to *Y*. Let *f* be randomly chosen from *F*. The probability of *f* being one-to-one is ______.
- $|X|=2, |Y|=20$
- Total functions possible $= |Y|^{|X|} = 20^2$
- One to one function possible $= {}^{|Y|}P_{|X|} = {}^{20}P_2 = \frac{20!}{(20-2)!} = 20*19$
- The probability of f being one to one is $= \frac{20*19}{20\times20} = \frac{19}{20} = .95$
- Ans : .95

- ***GATE CS 2015 Set 3 | Question: 23**
- Suppose U is the power set of the set $S=\{1,2,3,4,5,6\}$. For any $T \in U$, let $|T|$ denote the number of elements in T and T' denote the complement of T. For any $T, R \in U$ let $T \setminus R$ be the set of all elements in T which are not in R. Which one of the following is true?
- (A) $\forall X \in U, (|X| = |X'|)$ (B) $\exists X \in U, \exists Y \in U, (|X|=5, |Y|=5 \text{ and } X \cap Y = \emptyset)$
- (C) $\forall X \in U, \forall Y \in U, (|X|=2, |Y|=3 \text{ and } X \setminus Y = \emptyset)$ (D) $\forall X \in U, \forall Y \in U, (X \setminus Y = Y' \setminus X')$
- $S=\{1,2,3,4,5,6\}$
- A. Let $X=[\{1\},\{2\},\{1,3\}]$ therefore $|X'|=64-3=61$, $|X| \neq |X'|$, Wrong
- B. Let $X=\{1,2,3,4,5\}, Y=\{2,3,4,5,6\}$, $X \cap Y=\{2,3,4,5\}$
- Wrong as any two possible subsets can have some elements in common ,Hence, $X \cap Y$ cannot be always null.
- C.Let $X=\{2,5\}$,$Y=\{1,3,5\}$, $X \setminus Y=\{2\}$
- Wrong as it is not always $\emptyset$ .Sometimes it is $\emptyset$.
- D. $X \setminus Y = X \cap Y'$
- $Y' \setminus X' = Y' \cap (X')' = Y' \cap X = X \cap Y'$
- Ans : $(D) \forall X \in U, \forall Y \in U, (X \setminus Y = Y' \setminus X')$

- ***GATE CS 2015 Set 3 | Question: 41**
- Let R be a relation on the set of ordered pairs of positive integers such that $((p,q),(r,s)) \in R$ if and only if $p-s=q-r$. Which one of the following is true about R?
- (A)Both reflexive and symmetric (B)Reflexive but not symmetric
- (C)Not reflexive but symmetric (D)Neither reflexive nor symmetric
- Reflexive
- If ((p,q) , (p,q))∈R then p-q=q-p , not possible
- Symmetric
- If ((p,q),(r,s))∈R→ ((r,s),(p,q))∈R
- ((p,q),(r,s))∈R means p-s=q-r
- ((r,s),(p,q)) ∈R means r-q=s-p
- r-q=s-p ⇒ -(q-r)=-(p-s) ⇒ (q-r)=(p-s)
- Ans : (C)Not reflexive but symmetric

- ***GATE CS 2016 Set 1 | Question: 28**
- A function $f:N^+ \rightarrow N^+$, defined on the set of positive integers N^+, satisfies the following properties: $f(n)=f(n/2)$ if n is even
- $f(n)=f(n+5)$ if n is odd
- Let $R=\{i \mid \exists j: f(j)=i\}$ be the set of distinct values that f takes. The maximum possible size of R is ______________.
- Let $f(1)=x$,
- $f(2)=f(2/2)=x$
- $f(3)=f(3+5)= f(8)= f(4)=f(2)=x$
- $f(4)=f(2)=x$
- $f(5)=f(10)= f(5)=y$ let
- $f(6)=f(3)=x$
- So there are 2 distinct values that f takes.
- Ans : 2

- ***GATE CS 2016 Set 2 | Question: 26**
- A binary relation *R* on N×N is defined as follows: (a,b)R(c,d) if a ≤ c or b ≤ d. Consider the following propositions:
- *P*: *R* is reflexive. *Q*: *R* is transitive.
- Which one of the following statements is **TRUE**?
- (A)Both P and Q are true. (B)P is true and Q is false.
- (C)P is false and Q is true. (D)Both P and Q are false.
- P:Reflexive (a,b)R(a,b) a=a,b=b ,True
- Q:Transitive ((a,b)R(c,d)) , ((c,d)R(d,e)) then ((a,b)R(d,e))
- ((2,4)R(5,3)) , ((5,3)R(1,3)) then ((2,4)R(1,3)) not possible
- 2 ≤5 or 4 ≤3 , 5 ≤1 or 3 ≤3 2 ≤1 or 4 ≤3
- R is reflexive but not transitive
- Ans : (B)P is true and Q is false.

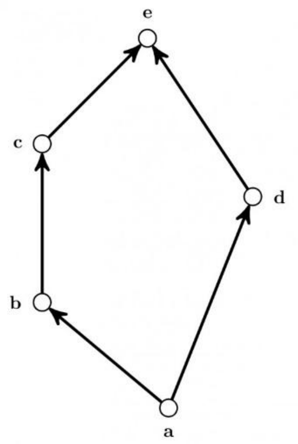
- ***GATE CS 2016 Set 2 | Question: 28**
- Consider a set U of 23 different compounds in a chemistry lab. There is a subset S of U of 9 compounds, each of which reacts with exactly 3 compounds of U. Consider the following statements:

I. Each compound in U \ S reacts with an odd number of compounds.
II. At least one compound in U \ S reacts with an odd number of compounds.
III. Each compound in U \ S reacts with an even number of compounds.

- Which one of the above statements is **ALWAYS TRUE**?
- (A)Only I (B)Only II (C)Only III (D)None.
- "\" is the set difference operation. Same as U – S.
- Since U is universal set, U\S would give complement of S = $\bar{S}$
- Let S contains Compounds numbered {1,2,3…8, 9} so U\S contains Compounds {10, 11, 12…. 22, 23}
- Consider these compounds to be vertices of a graph.
- An edge b/w two vertices indicate that the compounds react with each other.

- This graph has NO multiple edge,no directed edges cause if one compound reacts with other it also means other reacts with it too. Single edge represent reaction b/w both.It has NO Loops cause compound don't react with itself.
- Hence graph is simple undirected graph.
- "An undirected graph has even number of vertices of odd degree"
- 9 vertices of this graph have degree 3 (odd degree) cause 9 compounds react with 3 other compounds.
- Hence there must be at LEAST 1 more vertex which must have an odd degree.
- This extra compound must belong to U\S cause 9 compounds in S have already been accounted for.
- This implies statement II in the question is TRUE.
- Other 2 statements are False.
- Ans : (B)Only II

- ***GATE CS 2017 Set 2 | Question: 21**
- Consider the set X={a,b,c,d,e} under partial ordering R = {(a,a),(a,b),(a,c),(a,d),(a,e),(b,b),(b,c),(b,e),(c,c),(c,e),(d,d),(d,e),(e,e)}
- The Hasse diagram of the partial order (X,R) is shown below.
- The minimum number of ordered pairs that need to be added to R to make (X,R) a lattice is ______



- A Hasse Diagram is called a Lattice if, for every pair of elements, there exists a LUB and GLB.
- In the above Hasse Diagram, LUB and GLB exist for every two elements.
- So, it is already a Lattice.
- Ans : The Minimum number of ordered pairs that need to be added 0.

- ***GATE CS 2018 | Question: 19**
- Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is ______ .
- O(G)=84
- O(H)=2,3,4,6,7,12,14,21,28,42
- Ans : 42

- ***GATE CS 2018 | Question: 27**
- Let N be the set of natural numbers. Consider the following sets,
- P: Set of Rational numbers (positive and negative)
- Q: Set of functions from {0,1} to N
- R: Set of functions from N to {0,1}
- S: Set of finite subsets of N
- Which of the above sets are countable?
- A.Q and S only B.P and S only C.P and R only D.P,Q and S only
- P: Set of Rational numbers are ***countable***. Rational numbers are of the form p/q where p,q are integers.
- Q: Set of functions from {0,1} to N. There are N^2 such functions. Hence ***countable***.
- R: Set of functions from N to {0,1}. There are 2^N such functions. If a set S is countable, then P(S) i.e 2^S is uncountable.
- Hence, statement R is ***uncountable***.
- S: Set of finite subsets of N. They are ***countable***. Every subset of a countable set is either countable or finite.
- Ans : D.P,Q and S only

- ***GATE CS 2019 | Question: 5**
- Let $U=\{1,2,\ldots,n\}$ Let $A=\{(x,X)|x\in X, X\subseteq U\}$. Consider the following two statements on $|A|$.
- I. $|A|=n2^{n-1}$ II. $|A|=\Sigma_{k=1}^{n} k\binom{n}{k}$
- Which of the above statements is/are TRUE?
- (A)Only I (B)Only II (C)Both I and II (D)Neither I nor II
- For n=2, U={1,2}
- The subsets of U are {Φ,{1},{2},{1,2}}
- So , set X has 4 possibilities. x can be 1 or 2.
- A=(x,X) = (1,Φ),(1,{1}),(1,{2}),(1,{1,2}),(2,Φ),(2,{1}),(2,{2}),(2,{1,2})
- (1,Φ), (1,{2}),(2,Φ),(2,{1}) will not be considered, as x ∉ X.
- so A={ (1,{1}),(1,{1,2}),(2,{2}),(2,{1,2})}
- ∴ |A|=4
- If n=3,U={1,2,3},
- A={ (1,{1}),(1,{1,2}), (1,{1,3}), (1,{1,2,3}),(2,{2}),(2,{1,2}), (2,{2,3}), (2,{1,2,3}), (3,{3}),(3,{1,3}), (3,{2,3}), (3,{1,2,3})}
- ∴ |A|=12

- I. $|A|=n2^{n-1}$
- When n=2 ⇒ $2*2=4$
- When n=3 ⇒ $3*2^2=12$
- II. $|A|=\Sigma_{k=1}^{n} k\binom{n}{k}$
- When n=2 ⇒ $1*{}^2C_1+2*{}^2C_2=2+2=4$
- When n=3 ⇒ $1*{}^3C_1+2*{}^3C_2+3*{}^3C_3=3+2*3+3=12$
- *or*
- $U=\{1,2,\ldots,n\}$ Let $A=\{(x,X)|x\in X, X\subseteq U\}$.
- $|X|=2^n=\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\ldots\ldots\binom{n}{n}$
- x can be any element of individual subset X.
- $|A|=\binom{n}{0}*0+\binom{n}{1}*1+\binom{n}{2}*2+\ldots\ldots\binom{n}{n}*n=n\,2^{n-1}$
- Ans : (C)Both I and II

- ***GATE CS 2019 | Question: 10**
- Let G be an arbitrary group. Consider the following relations on G:
- $R_1: \forall a,b \in G, aR_1b$ if and only if $\exists g \in G$ such that $a=g^{-1}bg$
- $R_2: \forall a,b \in G, aR_2b$ if and only if $a=b^{-1}$
- Which of the above is/are equivalence relation/relations?
- A. R_1 and R_2 B. R_1 only C. R_2 only D. Neither R_1 nor R_2
- **Reflexive:** $a = g^{-1}ag$ can be satisfied by putting $g = e$, identity "e" always exists in a group.
- **Symmetric:** $aRb \Rightarrow a = g^{-1}bg$ for some g
- $\Rightarrow b = gag^{-1} = (g^{-1})^{-1}a(g^{-1})$ always exists for every $g \in G$.
- **Transitive:** aRb and bRc $\Rightarrow a = g_1^{-1}bg_1$ and $b = g_2^{-1}cg_2$ for $\exists\ g_1, g_2 \in G$.
- Now $a = g_1^{-1} g_2^{-1} cg_2g_1 = (g_2g_1)^{-1} cg_2g_1$
- $g_1 \in G$ and $g_2 \in G \Rightarrow g_2g_1 \in G$ since group is closed so aRb and bRc $\Rightarrow$ aRc
- R_1 is a equivalence relation, because it satisfied reflexive, symmetric, and transitive
- $\mathbf{R_2}$ is not equivalence because it does not satisfied reflexive condition of equivalence relation:
- $aR_2a \Rightarrow a = a^{-1}\ \forall a$ which not be true in a group.
- Ans : B. R_1 only

- ***GATE CS 2020 | Question: 17**
- Let *R* be the set of all binary relations on the set {1,2,3}. Suppose a relation is chosen from *R* at random. The probability that the chosen relation is reflexive (round off to 3 decimal places) is _______.
- Number of reflexive relation are = $2^{n^2 - n} = 2^{3^2 - 3} = 2^6$
- Number of relations possible $=2^{n^2} = 2^{3^2} = 2^9$
- The probability that the chosen relation is reflexive $= \frac{2^6}{2^9} = \frac{1}{2^3} = \frac{1}{8} = .125$
- Ans : .125

- ***GATE CS 2020 | Question: 18**
- Let G be a group of 35 elements. Then the largest possible size of a subgroup of G other than G itself is _______.
- O(G)=35
- O(H)=1,5,7,35
- Largest size of subgroup =7
- Ans : 7

- ***GATE CS 2021 Set 1 | Question: 34**
- Let G be a group of order 6, and H be a subgroup of G such that 1<|H|<6. Which one of the following options is correct?
- A. Both G and H are always cyclic
- B. G may not be cyclic, but H is always cyclic
- C. G is always cyclic, but H may not be cyclic
- D. Both G and H may not be cyclic
- Lagrange's Theorem: The order of every subgroup of G divides the order of G
- O(G)=6
- O(H)=2 or 3
- Any group of prime order is cyclic hence **H is always cyclic**
- **G may or may not be cyclic.**
- Ans: (B) **G may not be cyclic, but H is always cyclic**

- ***GATE CS 2021 Set 1 | Question: 43**
- A relation R is said to be circular if aRb and bRc together imply cRa.
- Which of the following options is/are correct?
- (A) If a relation *S* is reflexive and symmetric, then *S* is an equivalence relation.
- (B) If a relation S is circular and symmetric, then S is an equivalence relation.
- (C) If a relation S is reflexive and circular, then S is an equivalence relation.
- (D) If a relation S is transitive and circular, then S is an equivalence relation.
- **Equivalence relation : reflexive, symmetric and transitive**
- A. S is reflexive and symmetric it doesn’t satisfy equivalence property
- B. S is circular and symmetric means it can be transitive but not reflexive ,So not equivalence
- C. S is reflexive and circular hence can be symmetric and transitive , So equivalence
- D. S is both transitive and circular but not reflexive, hence not equivalence
- Ans:(C) If a relation S is reflexive and circular, then S is an equivalence relation.

- ***GATE CS 2021 Set 2 | Question: 11**
- Consider the following sets, where $n \geq 2$:
- S_1: Set of all $n \times n$ matrices with entries from the set $\{a,b,c\}$
- S_2: Set of all functions from the set $\{0,1,2,\ldots,n^2-1\}$ to the set $\{0,1,2\}$
- Which of the following choice(s) is/are correct?
- (A) There does not exist a bijection from S_1 to S_2
- (B) There exists a surjection from S_1 to S_2
- (C) There exists a bijection from S_1 to S_2
- (D) There does not exist an injection from S_1 to S_2
- S_1: $n \times n$ matrices contain n^2 elements ,3 choices for each element, so number of such matrices $=3^{n^2}$.
- S_2: number of functions possible $= 3^{n^2}$.
- $|S_1| = |S_2|$ So bijection is possible, surjection & injection is also possible.
- Ans : (B),(C)

- ***GATE CS 2021 Set 2 | Question: 50**

- Let S be a set of consisting of 10 elements. The number of tuples of the form (A,B) such that A and B are subsets of S, and A⊆B is ___________.
- Method 1:Lets S={a},|S|=1
- (A,B)= (Ø,Ø),(Ø,{1}),({1},{1}) number of tuples =3=3^1
- Lets S={a,b},|S|=2
- (A,B)= (Ø,Ø),(Ø,{1}),(Ø,{2}), (Ø,{1,2}), ({1},{1}),({1},{1,2}),({2},{2}),({2},{1,2}),({1,2},{1,2})
- Number of tuples =9=3^2
- |S|=10 , number of tuples =3^{10} =59049
- Method 2: We want the ordered pairs (A,B) where (A⊆S,B⊆S;A⊆B;)
- For every element x of set S, we have three choices
- Choice 1 : x∉A and x∉B ,Choice 2 : x∉A and x∈B,Choice 3 : x∈A and x∈B
- For each x we have 3 choices, and |S|=10 . So, answer will be 3^{10}.
- Ans : 59049

- ***GATE CS 2022 |Question: 17**
- Which of the following statements is/are TRUE for a group G?
- A. If for all x,y $\in G, (xy)^2 = x^2y^2$, then G is commutative.
- B. If for all $x \in G, x^2 = 1$, then G is commutative. Here, 1 is the identity element of G.
- C. If the order of G is 2, then G is commutative.
- D. If G is commutative, then a subgroup of G need not be commutative.
- A. $(xy)^2 = x^2y^2$
- $(xy)(xy)=(xx)(yy) \Rightarrow xyxy=xxyy$
- After cancellation of x from left side and y from right side yx=xy .True
- B. $\forall x \in G, x^2=1$ means every element is inverse of itself ,$x=x^{-1}$
- $(ab)^{-1}=b^{-1}a^{-1} \Rightarrow ab=ba$, hence commutative true .
- C. Order of G is 2, one element is Identity element, So another element x is inverse to itself. By Option B, G is commutative.True
- D. If G is commutative, then a subgroup of G should be commutative.False
- Ans :A,B,C

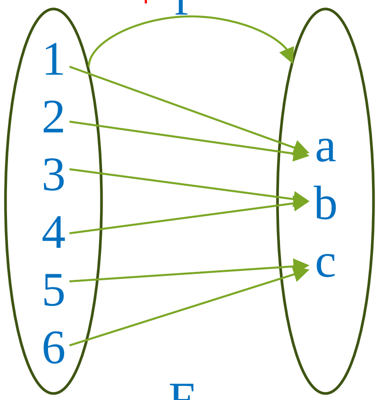
- ***GATE CS 2022 | Question: 26**
- Which one of the following is the closed form for the generating function of the sequence $\{a_n\}_{n\geq 0}$ defined below?
- $a_n = \{n+1, n \text{ is odd} \quad = \{1, \text{otherwise}$
- (A) $\frac{x(1+x^2)}{(1-x^2)^2} + \frac{1}{1-x}$ (B) $\frac{x(3-x^2)}{(1-x^2)^2} + \frac{1}{1-x}$ (C) $\frac{2x}{(1-x^2)^2} + \frac{1}{1-x}$ (D) $\frac{x}{(1-x^2)^2} + \frac{1}{1-x}$
- Generating function G(x) for the sequence a_n is $G(x) = \sum_{n=0}^{\infty} a_n x^n$
- The sequence $a_0x^0+a_1x^1+a_2x^2+a_3x^3+a_4x^4+a_5x^5+a_6x^6+\ldots\ldots\ldots\infty$
- $=1+2x+x^2+4x^3+x^4+6x^5+x^6+8x^7+x^8+10x^9+x^{10}+\ldots\ldots\ldots\infty$
- $=(1+x^2+x^4+x^6+x^8+x^{10}\ldots\ldots\infty)+(2x+4x^3+6x^5+8x^7+10x^9+\ldots\ldots\ldots\infty)$
- $1+x^2+x^4+x^6+x^8\ldots\ldots\infty=\frac{1}{1-x^2}$ Let $2x+4x^3+6x^5+8x^7+10x^9+\ldots\ldots\ldots\infty=P$
- $Px^2= 2x^3+4x^5+6x^7+8x^9+10x^{11}+\ldots\ldots\ldots\infty$
- $P-Px^2=2x+2x^3+2x^5+2x^7+2x^9+2x^{11}+\ldots\ldots\ldots\infty$
- $=2x(1+x^2+x^4+x^6+x^8+x^{10}+\ldots\ldots\ldots\infty)$
- $P-Px^2=\frac{2x}{1-x^2}$

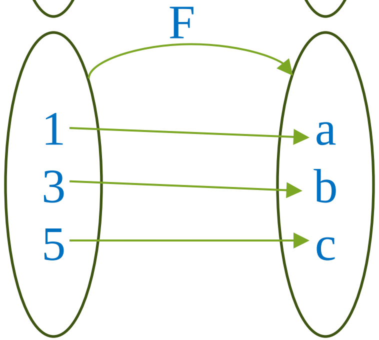
- $\Rightarrow P(1-x^2)=\frac{2x}{1-x^2} \Rightarrow P=\frac{2x}{(1-x^2)^2}$
- $=\frac{1}{1-x^2}+\frac{2x}{(1-x^2)^2}$
- $=\frac{1+x-x}{1-x^2}+\frac{2x}{(1-x^2)^2}$
- $=\frac{1+x}{1-x^2}-\frac{x}{1-x^2}+\frac{2x}{(1-x^2)^2}$
- $=\frac{1+x}{(1-x)(1+x)}-\frac{x}{1-x^2}+\frac{2x}{(1-x^2)^2}$
- $=\frac{1}{(1-x)}-\frac{x}{1-x^2}+\frac{2x}{(1-x^2)^2}=\frac{1}{(1-x)}+\frac{2x}{(1-x^2)^2}-\frac{x}{1-x^2}$
- $=\frac{1}{(1-x)}+\frac{2x-x(1-x^2)}{(1-x^2)^2}$
- $=\frac{1}{(1-x)}+\frac{2x-x+x^3}{(1-x^2)^2}=\frac{1}{(1-x)}+\frac{x+x^3}{(1-x^2)^2}$
- Ans : (A) $\frac{x(1+x^2)}{(1-x^2)^2}+\frac{1}{1-x}$

- ***GATE CS 2023 | Question: 39**
- Let f:A→B be an onto (or surjective) function, where A and B are nonempty sets. Define an equivalence relation ~ on the set A as

  $a_1 \sim a_2$ if $f(a_1)=f(a_2)$,
- where $a_1, a_2 \in A$. Let $\varepsilon=\{[x]:x\in A\}$ be the set of all the equivalence classes under ~. Define a new mapping F: $\varepsilon \rightarrow$B as F([x])=f(x), for all the equivalence classes [x] in ε.

  Which of the following statements is/are TRUE?
- (A)F is NOT well-defined. (B)F is an onto (or surjective) function.
- (C)F is a one-to-one (or injective) function. (D)F is a bijective function.





- Let's A={1,2,3,4,5,6},B={a,b,c}
- 1 ~2 ,3 ~4,5 ~6
- (A)False ,F is well-defined.
- (B)True ,F is an onto function as all elements of co-domains are mapped.
- (C)True ,F is a one-to-one function as all domains maps to different elements.
- (D)True , F is a bijective function as its both one-to-one and onto.
- Ans :(B) , (C) , (D)

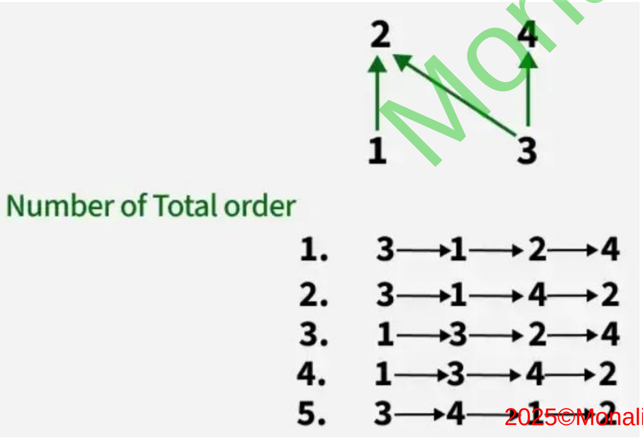
- ***GATE CS 2023 | Question: 41**
- Let X be a set and 2^X denote the powerset of X.
- Define a binary operation Δ on 2^X as follows: $A\Delta B=(A-B)\cup(B-A)$.
  Let $H=(2^X,\Delta)$. Which of the following statements about *H* is/are correct?
- (A) H is a group.
- (B) Every element in *H* has an inverse, but *H* is NOT a group.
- (C) For every $A\in 2^X$, the inverse of *A* is the complement of *A*.
- (D) For every $A\in 2^X$, the inverse of *A* is *A*.
- $A \Delta B = (A - B) \cup (B - A)=(A\cup B)-(A\cap B)$
- **Closure:** 2^X is the powerset of X, A Δ B will always be a subset of 2^X, so closure is satisfied.
- **Associativity:** union is associative , hence Δ satisfied this property
- **Identity Element:** The identity element e would be the empty set {}.
- $A \Delta \{\} = (A - \{\}) \cup (\{\} - A) = A \cup \{\} = A$. So, the identity element exists.
- **Inverse Element:** The inverse element would be A itself
- $A \Delta A = (A - A) \cup (A - A) = \{\} \cup \{\} = \{\}$ (the identity element).
- So, every element has an inverse.
- Ans : A,D

- ***GATE CS 2024 | Set 1 | Question: 22**
- Let A and B be non-empty finite sets such that there exist one-to-one and onto functions (i) from A to B and (ii) from A×A to A∪B. The number of possible values of |A| is ____________.
- Let $|A| = n$. Given that there exists a bijection between sets A and B, it follows that $|B| = n$.
- The number of elements in A × A is given by: $|A \times A| = n * n = n^2$
- There exists a bijection between A × A and A ∪ B. Thus: $|A \cup B| = n^2$
- $|A \cup B| \le |A| + |B|$
- $|A \cup B| \le n + n = 2n$
- $n^2 \le 2n$
- $n \le 2$
- Since n cannot be 0(As the sets are non-Empty) $n = 1$ or $n = 2$
- Maximum possible value=2
- Ans: 2

GATE CS 2024 | Set 1 | Question: 42

- Consider the operators ⋄ and □ defined by a⋄b=a+2b,a□b=ab, for positive integers. Which of the following statements is/are TRUE?
- A.Operator ⋄ obeys the associative law
- B.Operator □ obeys the associative law
- C.Operator ⋄ over the operator □ obeys the distributive law
- D.Operator □ over the operator ⋄ obeys the distributive law
- **# is associative if : (a # b) # c = a # (b # c).**
- **@ is distributive over # if a @ (b # c) = (a @ b) # (a @ c)**
- **A :** (a ◊ b) ◊ c = (a+2b) ◊ c = a + 2b + 2c
- a ◊ (b ◊ c) = a ◊ (b + 2c) = a + 2b + 4c
- (a ◊ b) ◊ c ≠ a ◊ (b ◊ c).
- **B:** (a □ b) □ c = (ab) □ c = abc.
- a □ (b □ c) = a □ (bc) = abc
- (a □ b) □ c = a □ (b □ c).
- **Ans:** (B) & (D)
- **C:** a ◊ (b □ c) = a ◊ (bc) = a + 2bc
- (a ◊ b) □ (a ◊ c) = (a + 2b) □ (a+2c) = a^2+2ac+2ab+4bc.
- a ◊ (b □ c) ≠ (a ◊ b) □ (a ◊ c).
- **D:** a □ (b ◊ c) = a □ (b+2c) = ab + 2ac
- (a □ b) ◊ (a □ c) = (ab) ◊ (ac) = ab + 2ac.
- a □ (b ◊ c) = (a □ b) ◊ (a □ c).

- ***GATE CS 2024 | Set 2 | Question: 24**
- Let P be the partial order defined on the set {1,2,3,4} as follows
  $P=\{(x,x) | x\in\{1,2,3,4\}\}\cup\{(1,2),(3,2),(3,4)\}$
- The number of total orders on {1,2,3,4} that contain P is ____________.
- If *(S, ≤)* is a poset and every two elements of *S* are comparable, *S* is called a *totally ordered* or *linearly ordered set*, and is called a *total order* or a *linear order.*
- 1→2 ,3 →2 , 3→4
- Total Order Set: [1,3,2,4]; [1,3,4,2]; [3,1,2,4]; [3,1,4,2]; [3,4,1,2]
- There will be a total of 5 total orders.
- Ans : 5



- ***GATE CS 2024 | Set 2 | Question: 53**
- Let Z_n be the group of integers {0,1,2,…,n−1} with addition modulo n as the group operation. The number of elements in the group $Z_2 \times Z_3 \times Z_4$ that are their own inverses is ___________.
- Z_2={0,1}
- 0 inverse=0 ,1 inverse=1
- The number of elements in Z_2 that are self inverses is 2
- Z_3={0,1,2}
- 0 inverse=0
- The number of elements in Z_3 that are self inverses is 1
- Z_4={0,1,2,3}
- 0 inverse=0 ,2 inverse=2
- The number of elements in Z_4 that are self inverses is 2
- The number of elements in the group $Z_2 \times Z_3 \times Z_4$ that are their own inverses is 2*1*2=4
- Ans : 4

+2	0	1
0	0	1
1	1	0

+3	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

+4	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2